BREXIT: A COMPARISON OF DYNAMIC VOTING GAMES WITH IRREVERSIBLE OPTIONS[⋆]

Benny Moldovanu[a], Frank Rosar[b,∗]

May 6, 2021

[a]*Department of Economics, Institute for Microeconomics, University of Bonn, 53113 Bonn, Germany*
[b]*Department of Economics, Institute for Microeconomics, University of Bonn, 53113 Bonn, Germany*

Abstract

A large polarized electorate decides repeatedly between a reversible alternative (REMAIN) and an irreversible alternative (LEAVE) in an environment where the aggregate short-term effects of the decision vary from period to period. Decisions by simple majority or by a too low supermajority may perform poorly under circumstances where it is socially optimal to never LEAVE, as they can exhibit equilibria where LEAVE is chosen quickly. In general, a too low supermajority rule can have much higher welfare costs than a too high supermajority rule. If REMAIN also becomes permanent when it wins by a large enough margin, and if a new vote is triggered otherwise, particularly poor performances of the simple majority rule are avoided. The large asymmetry in potential welfare costs disappears, and the majority requirement becomes a relatively less important instrument.

Keywords: Dynamic voting, Irreversible option, Option value, Supermajority rules, Voting by two-sided supermajority

JEL-Codes: D72, D82, C72

[⋆]Declarations of interest: None.
[∗]Corresponding Author. Email: email@frankrosar.de. Telephone: +49 228 73 6192. Fax: +49 228 73 7940.

1. Introduction

On 23 June 2016 the UK voted by referendum to leave the European Union. The referendum required a simple majority for LEAVE. While LEAVE is widely believed to be irreversible for the foreseeable future, maybe for a whole generation or more, REMAIN just defers the final decision and allows a new vote in the future—hence it is potentially reversible. Other Brexit-like decisions that were decided by simple majority were the 1995 Quebec-Referendum, and the 2014 Scottish independence referendum. The 2006 Montenegrin independence referendum used a 55% supermajority rule.

Should such fundamental decisions between two asymmetric options be made by a symmetric procedure like simple majority? To study this question, we introduce a tractable, dynamic framework of a polarized society who chooses among a reversible option (called here REMAIN) and an irreversible one (called here LEAVE). Our main findings are

1. In environments where it is sometimes optimal to LEAVE, the irreversible option LEAVE is adopted too easily. Supermajority rules can lead to better decisions.

2. In environments where it is never optimal to LEAVE, an equilibrium where LEAVE is chosen quickly can coexist with one where LEAVE is never chosen. Requiring a sufficiently large supermajority for LEAVE avoids the existence of the welfare-inferior equilibrium.

3. There is an asymmetry in the potential welfare costs from non-optimal rules: a too low supermajority rule can have a much higher cost than a too high supermajority rule.

4. Welfare-inferior equilibria can often be avoided without fine-tuning if a final decision for either alternative requires winning by a certain margin.

From an axiomatic (and static) perspective, there are reasons in favor of the simple majority rule (May, 1952, Dasgupta and Maskin, 2008). Most notably, supermajority rules do not respect the "one person-one vote" principle: they allow a minority to impose the status quo on a majority that may prefer change. In dynamic contexts where LEAVE is likely to be better in the future, this can be particularly harmful if it allows the initial status quo, REMAIN, to prevail for a very long time.

In our model, a large electorate repeatedly decides between REMAIN and LEAVE. The latter option ends the decision process. An agent knows in each period whether he is a LEAVE-winner or a LEAVE-loser from a short-term perspective, but faces uncertainty about future payoffs.

Whether an agent sees himself as a LEAVE-winner or a LEAVE-loser from a short-term perspective depends on the temporary issues, like a refugee crisis, a pandemic, a deep recession. We thus allow the mass of short-term LEAVE-winners to fluctuate over time, and we distinguish among two types of environments: the environment is REMAIN-friendly (LEAVE-friendly) if the probability of a future majority of LEAVE-winners is lower (higher) that 50%. The existence of only two types of agents models a simple notion of extreme polarization, but our results continue to hold in less polarized societies.[1]

We normalize our model such that, in the benchmark case where both decisions are reversible, it is socially optimal to choose the option preferred by a majority from a short-term perspective. Contrasting the benchmark case, REMAIN yields a positive **option value** when LEAVE is irreversible. Two main scenarios arise: If the environment is REMAIN-friendly, and if discounting is not too high, it is socially optimal to never LEAVE. Otherwise, it is socially optimal to LEAVE in the first period with a sufficiently large supermajority of LEAVE-winners.

An agent's preferred decision may differ from the social planner's preferred decision for two reasons. Firstly, polarized agents have a more extreme view about the short-term effects whereas the planner weighs aggregate effects. Secondly, an agent who expects sub-optimal future decisions underestimates the option value of REMAIN.

Suppose first that it is socially optimal to never LEAVE and that simple majority is used. If an agent believes that the other agents vote myopically, he might expect that LEAVE will nevertheless be soon chosen. But, then the agent perceives the future consequences of LEAVE and of REMAIN as almost equal, and it may be thus individually optimal to vote myopically. A welfare-inferior equilibrium where LEAVE is quickly chosen often coexists with a welfare-optimal equilibrium where LEAVE is never chosen. This can be the case even though LEAVE-

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[1]Fueled by the rise of the internet and social media, polarization became increasingly important during the last two decades (Sunstein, 2018).

winners agree with LEAVE-losers that it would be optimal to never LEAVE, and even if all agents perceive the future as arbitrarily more important than the present.

The welfare-inferior equilibrium generally ceases to exist if a sufficiently large supermajority is required for LEAVE. Intuitively, under a supermajority rule, LEAVE is, on average, chosen at a later point in time. This makes it harder to support the belief that the future consequences of LEAVE and of REMAIN are similar.

Suppose next that the mass of current LEAVE-winners determines which decision is socially optimal. Myopic voting constitutes then a unique equilibrium for any supermajority rule that is not too large. Decisions by simple majority fail to reflect the option value of REMAIN. Supermajority rules allow to correct this failure by shifting the pivotal agent in the direction that is less eager to LEAVE.

In practice it is not always clear which precise game is being played. For example, the leader of the UK Independence party, Nigel Farage, a major Brexit proponent, warned shortly before the referendum that he would fight for a second referendum if the REMAIN campaign won by a narrow margin:

> "In a 52%-48% referendum this would be unfinished business by a long way. If the remain campaign win two-thirds to one-third that ends it."[2]

In view of such possibilities, we next analyze the game where the decision process ends as soon as either REMAIN or LEAVE wins by an a priori defined margin; if neither alternative wins by this margin, the vote is repeated in the next period. Even though voting at each stage is binary, such a mechanism effectively creates three social alternatives: LEAVE, REMAIN for now but vote again next period, and REMAIN forever.

For any margin required by a final decision, the myopic equilibrium in environments where it is socially optimal to never LEAVE and all agents view the future as arbitrarily more important than the present disappears now: LEAVE is then never chosen in equilibrium. A rough intuition is the following: If the agents expect that LEAVE might nevertheless be chosen in the future,

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[2]"Nigel Farage wants second referendum if Remain campaign scrapes narrow win", Mirror, 16 May 2016.

there exist circumstances under which REMAIN will win by a large enough margin to be logged it. This expectation drives in turn a wedge in the agents' assessment of REMAIN and LEAVE. If the future is important enough, this wedge is large and even LEAVE-winners always have an incentive to vote for REMAIN. However, there is still a role for supermajority rules if it is sometimes optimal to LEAVE. Requiring an optimally chosen supermajority for LEAVE implements then the socially optimal policy.

1.1. Related Literature

The classical literature on investment studies a single agent's trade-off between committing to an irreversible decision and waiting (see Dixit and Pindyck (1994) for a review). Instead, our focus here is on a situation where a group of agents collectively decides via voting.

In the collective search literature, a committee observes a stream of alternatives (candidates, proposals, …) and search ends when an alternative is accepted by a certain majority.[3] Recent contributions to this literature (Albrecht et al., 2010, Compte and Jehiel, 2010, Moldovanu and Shi, 2013) focused on the nature of the accepted proposals for exogenously fixed decision rules. The payoff structure in that literature resembles the one in a bargaining problem where the accepted alternative generates a one-time payoff. Instead, our payoff structure resembles a repeated game where stage payoffs are generated in each period. This difference in the payoff structure enables the emergence of our most interesting scenario: specifically, in the collective search literature, it is optimal from a welfare perspective to immediately accept any sufficiently strong alternative, whereas here it can be optimal to reject the irreversible alternative forever.

Another strand of literature investigates a group's decision to directly adopt a project, or to wait and decide about adopting a project after new information arrives. Gersbach (1993b) illustrates that for decisions by simple majority the option to wait can have a negative value (!). Gersbach (1993a) compares the case where new information arrives before the second period decision with that where it does not (or is ignored). He structures the possible cases where obtaining new information has a positive/negative value for a set or a majority of voters and how

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[3]This literature was initiated by Sakaguchi (1973) who analyzed equilibrium existence for two players under the unanimity rule. Kurano et al. (1980) extend the analysis to more players and to general majority rules. Ferguson (2005) shows that multiple stationary cutoff equilibria can exist in these models.

commitment to having or not having future decisions could help to solve the problem. Messner and Polborn (2012) show that a supermajority rule may be optimal and that the option to wait can have a negative value even if the optimal majority rule is employed. However, by restricting attention to a two-period model with i.i.d. stage payoffs and without discounting, Messner and Polborn implicitly assumed that present and future are equally important. Allowing the future to be more important than the present (while avoiding last round effects) as we do, adds qualitatively new insights.

For a class of dynamic collective decision games with a unique inefficient equilibrium, Roessler et al. (2018) study the following question: If agents ex ante have the opportunity to collectively commit to a policy that is better than the policy induced by the unique equilibrium, can they agree to do so? Under a power consistency condition that links the political power in the ex ante problem with that in the dynamic game, they find that such a commitment opportunity has no value. Both policies would be part of a Condorcet cycle. By contrast, when the welfare-inferior equilibrium coexists with the welfare-optimal equilibrium in our majority voting game, all agents would love to commit ex ante to the welfare-optimal policy. Responsible for this difference is the infinite repetition of our voting problem which allows for multiple equilibria.

Fernandez and Rodrik (1991) analyze the dynamic choice between the status quo (say, REMAIN) and a reform (LEAVE). Under simple majority voting they identify a **status quo bias** if the reform is reversible and if new information about the relative benefits of the reform only becomes available if it is adopted. Intuitively, an adopted reform that turns out to be ex-post suboptimal will be eventually repealed. In contrast, an initially not adopted reform will also not be adopted later because no new information that might change the voting incentives arrives. The bias in our paper goes into the opposite direction: The main force behind the status quo bias—the possibility to repeal the reform ex-post—disappears if the reform is irreversible. Moreover, if new (possibly limited) information becomes available even if the reform is initially not undertaken, an option value effect arises and causes a bias against the status quo.

Some related effects appear also in the literature on pre-commitment in environments where preferences can change over time. For example, in Tabellini and Alesina (1990), the initial

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median voter expects that the decisions in the next period will be sometimes taken by a new median voter with different preferences. As in our paper, this renders him excessively short-sighted and too eager to take irreversible actions.[4]

2. The Model

In each period $t = 1, 2, \ldots$ there are two possible decisions: REMAIN ($d_t = R$) and LEAVE ($d_t = L$). Once taken, LEAVE is irreversible. The stream of decisions $d_1, d_2, \ldots$ affects a continuum of risk-neutral agents having mass 1. In each period t, each agent i obtains a stage payoff of $\pi_t^i \in \{0, 1\}$ from LEAVE and a stage payoff of $\frac{1}{2}$ from REMAIN. If $\pi_t^i = 1$ (0), then agent i is a LEAVE-winner (loser) from a short-term perspective. In Extension I in Appendix A we explain how our results extend to a version of our model that relies on a less extreme notion of polarization where the stage payoff from LEAVE, π_t^i, can assume any value in $[0, 1]$. Agents discount future stage payoffs by a discount factor $\delta \in (0, 1)$.

The mass of LEAVE-winners, $p_t \in [0, 1]$, is independently and identically distributed across periods according to a c.d.f F with p.d.f f and full support. Conditional on p_t, the payoffs π_t^i are independently distributed across periods and across agents: Agent i is a LEAVE-winner with probability p_t and a LEAVE-loser with probability $1 - p_t$. In Extension II in Appendix A we allow for a serial correlation of individual stage payoffs across time. Let $\bar{p} \equiv \mathbb{E}[p_t]$. We call the environment *LEAVE-friendly* if $\bar{p} > \frac{1}{2}$, *REMAIN-friendly* if $\bar{p} < \frac{1}{2}$, and *neutral* if $\bar{p} = \frac{1}{2}$.

The timing within a period t is as follows: First, nature draws the mass of LEAVE-winners p_t, and reveals it publicly.[5] Second, nature privately reveals to each agent i whether he is a LEAVE-winner or loser. Third, if the previous decision was REMAIN, a new decision d_t is taken. Otherwise, $d_t = L$. Finally, stage payoffs realize.

The planner is utilitarian: her stage payoff is the average stage payoff across agents, i.e. it is p_t from LEAVE and $\frac{1}{2}$ from REMAIN, and she discounts future stage payoffs by δ.

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[4]Grüner (2017) approaches such a problem from a mechanism design perspective and finds a role for super-majority rules. In Battaglini and Harstad (2020), the current decision maker uses pre-commitments to increase the probability of staying in office by becoming more attractive for the next median voter.

[5]Whether p_t is observable will only matter for our analysis of two-sided majority voting in Section 5 and of Extension II in Appendix A. A motivation for the observability of p_t could be that experts are able to assess the consequences of LEAVE on the population from a short-term perspective and announce these publicly.

3. Centralized Decision Making

If LEAVE is also reversible, the decisions in different periods are not connected. The optimal policy follows from comparing the current stage payoffs p_t and $\frac{1}{2}$. At each stage, it is optimal to take the decision preferred by a majority from a short-term perspective.

If LEAVE is irreversible, the future consequences of the current decision matter. We can denote the planner's value of entering a period t with decision d by V_d^*. If the previous decision was REMAIN, her payoff from LEAVE is $p_t + \delta V_L^*$, and her payoff from REMAIN is $\frac{1}{2} + \delta V_R^*$. The Bellman equations for the planner's optimal policy are

$$\begin{cases} V_L^* = \mathbb{E}[p_t + \delta V_L^*] \\ V_R^* = \mathbb{E}[\max\{p_t + \delta V_L^*, \frac{1}{2} + \delta V_R^*\}] \end{cases}.$$

The values V_d^*, $d \in \{R, L\}$, are uniquely defined. The difference $\Delta^* \equiv \delta(V_R^* - V_L^*)$ represents the *option value* of the reversible decision, REMAIN. A **cutoff policy** with cutoff $p \in \mathbb{R}$ selects LEAVE if $p_t \geq p$ and selects REMAIN otherwise.[6] It follows from the Bellman equations that the cutoff policy with cutoff

$$p^* \equiv \tfrac{1}{2} + \Delta^*$$

is optimal, where Δ^* is the unique solution to

$$\Delta^* = \delta \mathbb{E}[\max\{0, \Delta^* + \tfrac{1}{2} - p_t\}]. \tag{1}$$

Obviously, $\Delta^* > 0$. Relative to the case where LEAVE is reversible, the planner is biased towards REMAIN: LEAVE is optimal only if there is a sufficiently large supermajority of LEAVE-winners.

Lemma 1. *Let*

$$\delta^* \equiv \frac{1}{2(1 - \bar{p})}. \tag{2}$$

Then $p^* \in (\frac{1}{2}, 1)$ for $\delta < \delta^*$, $p^* = 1$ for $\delta = \delta^*$, and $p^* > 1$ for $\delta > \delta^*$.

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[6]Allowing for cutoffs outside the interval $[0, 1]$ will be useful for expositional reasons although all cutoffs $p > 1$, and all cutoffs $p \leq 0$ describe the same policy, respectively. Specifically, p^* as defined by the next equation in the text can exceed 1.

The bias is so large that it is optimal to never LEAVE if and only if the environment is REMAIN-friendly ($\bar{p} < \frac{1}{2}$) and the future is important enough ($\delta \geq \delta^*$). This case seems relevant for applications with significant long-consequences, and it will give rise to interesting effects.

It will be useful for our analysis of referenda to understand how the planner compares non-optimal cutoff policies. For any cutoff p, consider the system of linear equations

$$\begin{cases} V_L(p) = \bar{p} + \delta V_L(p) \\ V_R(p) = F(p)(\frac{1}{2} + \delta V_R(p)) + (1 - F(p))(\mathbb{E}[p_t | p_t \geq p] + \delta V_L(p)) \end{cases}. \tag{3}$$

The system possesses a unique solution. $\delta V_d(p)$ describes the planner's continuation value from decision $d \in \{L, R\}$ if future decisions are taken according to cutoff policy p. It follows from (3) that

$$\Delta(p) \equiv \delta(V_R(p) - V_L(p)) = \frac{\delta}{1 - \delta F(p)} \int_0^p (\tfrac{1}{2} - p_t) \mathrm{d}F(p_t). \tag{4}$$

The utilitarian welfare from cutoff policy p is $V_R(p)$. Since the consequences of LEAVE are exogenous, maximizing $V_R(p)$ is equivalent to maximizing the future advantage of REMAIN over LEAVE, $\Delta(p)$. Employing $\Delta(p)$ for welfare comparisons will be convenient as it will also determine voting incentives. Lemma B.1 in Appendix B derives properties of $\Delta(p)$ that we will use in subsequent proofs.

4. Majority Voting with One-Sided Irreversible Decisions

Voting mechanism. Consider the voting game that is induced when decisions are taken by (super)majority with cutoff $\kappa \in [\frac{1}{2}, 1)$: In every period t such that the previous decision was REMAIN, each agent learns his stage payoff and then all agents simultaneously vote for LEAVE or for REMAIN. The decision is LEAVE if the mass of LEAVE-votes, l_t, is at least κ.

Equilibrium notion. We focus on Markov strategies that only condition on payoff-relevant information.[7] An equilibrium is a profile of symmetric Markov strategies such that, after any

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[7]Informally, an agent does not condition his behavior on parts of his private information/public history that are irrelevant if he believes that his vote is pivotal, and that other agents do not condition their behavior upon this information.

history where the previous decision was REMAIN, no agent has an incentive to unilaterally deviate under the assumption that his vote is pivotal.[8],[9]

An agent only conditions his behavior on whether he currently is a LEAVE-winner.[10] Let $\lambda(\pi_t^i)$ denote the probability with which an agent of type $\pi_t^i \in \{0, 1\}$ votes for LEAVE. A Markov strategy is then described by $(\lambda(0), \lambda(1)) \in [0, 1]^2$.

If all agents vote according to the Markov strategy $(\lambda(0), \lambda(1))$, the decision is

$$
d(p_t) = \begin{cases} L & \text{if } p_t \lambda(1) + (1 - p_t)\lambda(0) \geq \kappa \\ R & \text{if } p_t \lambda(1) + (1 - p_t)\lambda(0) < \kappa \end{cases} . \tag{5}
$$

Consider the system of linear equations

$$
\begin{cases} V_L = \mathbb{E}[\pi_t^i + \delta V_L] \\ V_R = \mathbb{E}[\mathbf{1}_{d(p_t)=L}(\pi_t^i + \delta V_L) + \mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} + \delta V_R)] \end{cases} \tag{6}
$$

implied by policy (5). It possesses a unique solution. δV_d describes the continuation value from decision $d \in \{R, L\}$. This continuation value is common to all agents due to the serial independence of individual stage payoffs.

Agent i's payoff from LEAVE (REMAIN) in the period t is $\pi_t^i + \delta V_L$ $(\tfrac{1}{2} + \delta V_R)$. The profile where each agent votes according to the Markov strategy $(\lambda(0), \lambda(1))$ forms an equilibrium if, and only if, this Markov strategy is optimal under pivotal voting given the future of advantage of REMAIN that it generates, $\Delta \equiv \delta(V_R - V_L)$. Formally:

$$
\begin{cases} \lambda(\pi_t^i) = 1 & \text{if } \pi_t^i - \tfrac{1}{2} > \Delta \\ \lambda(\pi_t^i) \in [0, 1] & \text{if } \pi_t^i - \tfrac{1}{2} = \Delta \\ \lambda(\pi_t^i) = 0 & \text{if } \pi_t^i - \tfrac{1}{2} < \Delta \end{cases} , \pi_t^i \in \{0, 1\}. \tag{7}
$$

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[8]This equilibrium notion is a refinement of Markov perfect equilibrium. In the version of our model with a large finite electorate, pivotality considerations are effective, and the same policies are essentially implementable in both versions. The continuum version simplifies the exposition.

[9]*Pivotal voting* also corresponds here to *sincere voting* where an agent's vote is sincere if he votes for the decision he prefers, taking as given the future decisions implied by the other agents' behaviors.

[10]If agent i is pivotal, the decision is LEAVE (REMAIN) if he votes LEAVE (REMAIN). The payoffs he assigns to the two options depend on his current stage payoff from LEAVE, π_t^i, and on the continuation values. If other agents do not condition on the public history, these values do not depend on it, and π_t^i is the only payoff-relevant information.

A policy is *implementable by κ-majority voting* if κ-majority voting possesses an equilibrium that induces this policy. A policy is *uniquely implemented by κ-majority voting* if it is the only policy that is implementable by κ-majority voting.

Equilibrium Analysis. Consider first our benchmark case where also LEAVE is reversible. Then, instead of (6), $V_R = V_L$ and Δ equals zero. The decision is LEAVE if the mass of LEAVE-winners, p_t, is at least κ. The optimal policy is uniquely implemented by the simple majority rule where $\kappa = \frac{1}{2}$.

If LEAVE is irreversible, LEAVE-winners have, for any given Δ, a stronger incentive to vote for LEAVE than LEAVE-losers, and we obtain:

Lemma 2. *Only cutoff policies are implementable by κ-majority voting. If cutoff policy p is implemented, then $\Delta = \Delta(p) = \frac{\delta}{1-\delta F(p)} \int_0^p (\frac{1}{2} - p_t) \, dF(p_t)$.*

We next explain the conditions under which a cutoff policy p is implementable by κ-majority voting: Fix any p and assume that the agents believe that future decisions are taken according to cutoff policy p. If $\Delta(p) \in (-\frac{1}{2}, \frac{1}{2})$, voting is myopic, and the cutoff policy $p = \kappa$ is induced. Hence, the cutoff policy p with $\Delta(p) \in (-\frac{1}{2}, \frac{1}{2})$ is implementable if and only if $p = \kappa$. If $\Delta(p) > \frac{1}{2}$, all agents vote REMAIN, and the induced decision is described by any cutoff policy $p > 1$. A cutoff policy p with $\Delta(p) > \frac{1}{2}$ is implementable if and only if $p > 1$. Lastly, if $\Delta(p) = \frac{1}{2}$, LEAVE-winners are indifferent, and may vote for LEAVE with any probability $\lambda(1) \in [0,1]$, whereas LEAVE-losers vote for REMAIN. Cutoff policy p with $\Delta(p) = \frac{1}{2}$ is implementable if and only if $p \geq \kappa$. The case where $\Delta(p) \leq -\frac{1}{2}$ leads to analogous implementability conditions.

Example. *For illustrations, we use the power distribution functions*

$$F_\gamma(p_t) \equiv \begin{cases} p_t^\gamma & \text{if } \gamma \in [1, \infty) \\ 1 - (1 - p_t)^{1/\gamma} & \text{if } \gamma \in (0, 1) \end{cases}.$$

Since $\bar{p} \equiv \frac{1}{2} + \frac{1}{2}\frac{\gamma-1}{\gamma+1}$, the environment is LEAVE-friendly if $\gamma > 1$, neutral if $\gamma = 1$ and REMAIN-friendly if $\gamma \in (0, 1)$.

Figure Ia (Figure Ib) illustrates the implementability conditions for a REMAIN-friendly (LEAVE-friendly) environment. In each panel, the black solid curve depicts $\Delta(p)$ for $\delta = 0.8$ and the black dashed curve depicts $\Delta(p)$ for $\delta = 0.9$. The black dots indicate (p^*, Δ^*). The gray
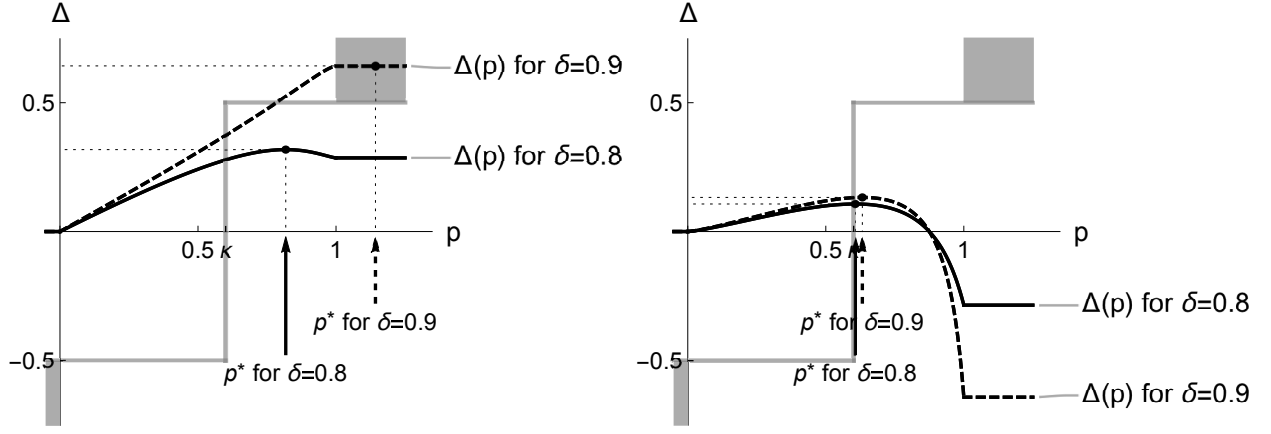
(a) REMAIN-friendly environment $[\gamma = \frac{3}{4}]$ (b) LEAVE-friendly environment $[\gamma = \frac{4}{3}]$

Figure I: Implementability conditions under κ-majority voting $[\kappa = 0.6]$

Implementable polices: Policies where the gray correspondence and the black curve intersects. All cutoffs $p > 1$ and all cutoffs $p \leq 0$ describe the same policy, respectively.

correspondence displays for any Δ the cutoff policies p that are consistent with pivotal voting under κ-majority voting. The intersections of the black curves and the gray correspondence describe the implementable policies.

We address now our main questions: How does the importance of the future, relative to the present, affect the set of implementable cutoff policies? Which (super)majority rule is optimal?

Proposition 1. *Consider a REMAIN-friendly environment. Let $\delta^* = \frac{1}{2(1-\bar{p})}$. Define $\delta^M \in (\delta^*, \infty)$ by $\delta^M \equiv (2\int_0^{1/2}(1-p_t)dF(p_t))^{-1}$.*

Case i: *$\delta \in (0, \delta^*)$. For all $\kappa \in [\frac{1}{2}, 1)$, κ-majority voting uniquely implements cutoff policy κ. Only the supermajority rule $\kappa = p^*$ implements the optimal policy.*

Case ii: *$\delta^M < 1$ and $\delta \in (\delta^*, \delta^M]$ or $\delta^M \geq 1$ and $\delta \in (\delta^*, 1)$. Then, $\Delta^{-1}(\frac{1}{2}) \in [\frac{1}{2}, 1)$. For all $\kappa \in [\frac{1}{2}, \Delta^{-1}(\frac{1}{2})]$, the optimal policy $p^* > 1$, which corresponds to never choosing LEAVE, and cutoff policy κ are both implementable by κ-majority voting. For all $\kappa \in (\Delta^{-1}(\frac{1}{2}), 1)$, κ-majority voting uniquely implements the optimal policy.*

Case iii: *$\delta^M < 1$ and $\delta \in (\delta^M, 1)$. For all $\kappa \in [\frac{1}{2}, 1)$, κ-majority voting uniquely implements the optimal policy.*

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If $\delta < \delta^*$ (Case i), the short-term effects dominate any long-term effects. Voting is myopic for any (super)majority rule. Since myopic voting fails to reflect the option value of REMAIN, such a consideration can only come through the voting rules. A supermajority becomes optimal.

If $\delta > \delta^*$ (Cases ii and iii), the option value of REMAIN is large, and it is optimal to REMAIN under any circumstances. The planner, LEAVE-losers and LEAVE-winners all prefer "REMAIN forever" over LEAVE. If agents believe that LEAVE will not be chosen in the future, all agents vote REMAIN under any (super)majority rule and the optimal policy is implemented. This welfare-optimal equilibrium can coexist with a welfare-inferior equilibrium where agents vote myopically: if they believe that REMAIN today will lead to LEAVE soon, the perceived future advantage of REMAIN can be small enough to turn myopic voting into optimal; conversely, the expectation of myopic voting in the future can support the belief that REMAIN today leads to LEAVE soon.

In Case ii, a myopic equilibrium exists for the simple majority rule, but it can be avoided by using a sufficiently large supermajority rule. Intuitively, a large enough supermajority leads to a later adoption of LEAVE, and a belief that REMAIN today leads to LEAVE in the near future cannot be supported. While the necessary supermajority may be large, smaller supermajorities are still useful since they improve the policy induced by the welfare-inferior equilibrium. In Case iii, the future is so important that, even under the simple majority rule, the belief about myopic voting in the future does not rationalize myopic voting today.

Example. *In Case i, the implementability conditions look like those for $\delta = 0.8$ and in Case ii like those for $\delta = 0.9$ in Figure Ia;[11] in Case iii, the shape of $\Delta(p)$ is similar to that in Case ii, but $\Delta(p)$ is so steep that it exceeds $\frac{1}{2}$ already at $p = \frac{1}{2}$. Consequently, the gray correspondence and the black curve do not intersect for $p < 1$. Figure II illustrates the δ-intervals where the three cases apply. If γ is close enough to 1, then Case iii never applies for $\delta \in (0, 1)$, i.e., $\delta^M \geq 1$ is possible.*

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[11]The figure shows that a third equilibrium exists in Case ii if $\kappa < \Delta^{-1}(\frac{1}{2})$. In this equilibrium, LEAVE-winners vote only with a certain probability for LEAVE, and an intermediate cutoff policy is implemented. This renders this equilibrium welfare-superior relative to the welfare-inferior pure strategy equilibrium, but welfare-inferior relative to the welfare-optimal pure strategy equilibrium. Also this equilibrium ceases to exist if a sufficiently large supermajority rule is used.
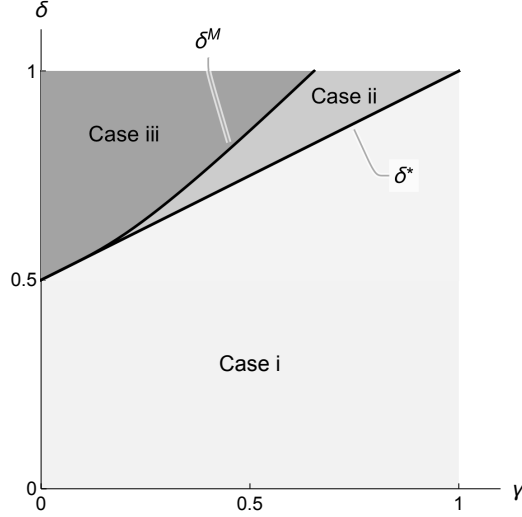
Figure II: δ-regions in Proposition 1 for power distribution functions

Corollary 1. *There exist REMAIN-friendly environments such that, for all $\delta \in (0,1)$, either Case i or Case ii of Proposition 1 applies. For such environments, a policy is implementable by the simple majority rule that leads to a welfare loss relative to the policy implemented by the optimal supermajority rule, $V_R(p^*) - V_R(\frac{1}{2})$, that grows without bound as $\delta \to 1$.*

In LEAVE-friendly and in neutral environments there always exist circumstances under which it is optimal to LEAVE. Supermajority rules are then necessary to incorporate the option value of REMAIN.

Proposition 2. *Consider a LEAVE-friendly or neutral environment.*

Case i: $\delta \in (0, \frac{1}{2\bar{p}})$. *For all $\kappa \in [\frac{1}{2}, 1)$, κ-majority voting uniquely implements cutoff policy κ. Only the supermajority rule $\kappa = p^*$ implements the optimal policy.*

Case ii: $\delta \in (\frac{1}{2\bar{p}}, 1)$. *Then, $\Delta^{-1}(-\frac{1}{2}) \in (p^*, 1)$. For all $\kappa \in [\frac{1}{2}, \Delta^{-1}(-\frac{1}{2})]$, κ-majority voting uniquely implements cutoff policy κ. For all $\kappa \in (\Delta^{-1}(-\frac{1}{2}), 1)$, κ-majority voting uniquely implements cutoff policy $\Delta^{-1}(-\frac{1}{2}) \in (p^*, 1)$. Only the supermajority rule $\kappa = p^*$ implements the optimal policy.*

Example. *In Case i of Proposition 2, the implementability conditions look like those for $\delta = 0.8$ and in Case ii like those for $\delta = 0.9$ in Figure Ib. Observe that $\Delta(p)$ is negative if p is close enough to 1.*

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A qualitative difference relative to Case i in Proposition 1 occurs only if the future is very important, and if a supermajority rule that is sufficiently much larger than the optimal rule is used. Then, although LEAVE is likely to be individually (and socially) optimal in the future, agents expect that it will take long until LEAVE is chosen after REMAIN. This gives LEAVE-losers an incentive to vote for LEAVE. In the unique equilibrium, each LEAVE-loser votes with a certain probability for LEAVE, limiting the welfare consequences of choosing too large a supermajority rule:

Corollary 2. *In any LEAVE-friendly environment, a larger than optimal supermajority rule leads to a welfare loss that converges to a value smaller than 1 as $\delta \to 1$.*

The Corollaries 1 and 2 point to an asymmetry in the welfare costs resulting from too low and too high supermajority rules. A too low supermajority can have much higher costs.

Fixing the voting rules before knowing details. Voting rules that are used for important decisions are not supposed to change often. Suppose there is an initial stage in which the planner fixes the majority rule κ while she only knows that the distribution F will be drawn from some class of distributions. Then, F is drawn and the κ-majority voting game that we analyzed in this section is played. Because of the general optimality of some supermajority rule in our base model, some supermajority rule will maximize the ex ante expected welfare in the extended model.

Binary nature and normalization of stage payoffs. Our analysis relied on two assumptions: The stage payoff from LEAVE was binary (our extreme notion of polarization) and the two possible realizations were equidistant from the stage payoff from REMAIN.

Without equidistant payoffs, sub- and supermajority rules can be optimal in the benchmark case. Since submajorities are typically not feasible for political reasons, deciding by simple majority is the best feasible option if the planner prefers a submajority rule. Our results translate easily to a setting with binary, non-equidistant payoffs: The planner's preference for a supermajority rule translates into a preference for a majority rule that is higher than the majority she prefers in the benchmark case.

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Stage payoffs need not to be binary, but polarization is crucial for our results. Intuitively, our analysis applies when the median agent prefers "more extreme" decisions than the planner. We explain this in more detail in Extension I in Appendix A.

Inter-temporal correlations. The assumption of uncorrelated short-term and long-term payoffs simplified our analysis. It was responsible for all voters and the social planner assessing the future effects of any policy in the same way. Supermajority rules were useful to improve the decisions that result from myopic voting behavior or to avoid that agents have incentives to vote myopically in the first place. Intuitively, inter-temporal correlations increase the agents' incentives to vote myopically and thus ceteris paribus (i.e., if they are introduced in a way that does not alter optimal policy) render supermajority rules even more important:[12] In environments where under the simple majority rule a welfare-optimal equilibrium coexists with a welfare-inferior equilibrium in our base model, the welfare-inferior equilibrium can become the unique equilibrium. We discuss this in more detail in Extension II in Appendix A.

5. Majority Voting with Two-Sided Irreversible Decisions

In his "unfinished business" quote which we referred to in the introduction, Nigel Farage warned that he would fight for a second referendum if the REMAIN campaign won by a narrow margin. While this quote highlights the reversibility of REMAIN, it suggests at the same time that REMAIN may be permanent if it wins by a sufficiently large margin. If this was the game, how would the agents' voting and the planner's design incentives change?[13]

Voting mechanism. Consider the game where the decision process ends if **either** LEAVE or REMAIN gains a κ-majority, $\kappa \in (\frac{1}{2}, 1)$. The decision is LEAVE (L) if the mass of LEAVE-votes l_t is at least κ, and it is REMAIN FOREVER ($R\infty$) if l_t is at most $1 - \kappa$; if $l_t \in (1 - \kappa, \kappa)$, the decision is REMAIN (for now) with a new vote in the next period (R). We call this mechanism *two-sided κ-majority voting*.[14]

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[12]Note that the ceteris paribus qualifier excludes the special case where individual stage payoffs are perfectly correlated over time. If stage payoffs are persistent, doing whatever the majority prefers from a short-term perspective is socially optimal and the simple majority rule uniquely implements the optimal policy.

[13]We thank an anonymous referee for pointing out this interpretation of the present voting game.

[14]A related mechanism is used in papal enclaves: Voting is repeated until a candidate obtains a certain

We require, for convenience, that the same supermajority makes either decision permanent. In practice, the supermajority required to end the process after REMAIN is exogenous, whereas the supermajority required for LEAVE is more of a design element. Our qualitative results in this section rely only on the fact that REMAIN can also become permanent, on the existence (but not the size) of a margin where the result is considered "too close" and where a new vote is triggered, and on the majority required for LEAVE.[15]

Generalization of the equilibrium notion. Two scenarios where an agent can be pivotal now arise: he can be pivotal for LEAVE and for REMAIN reaching the κ-majority. In the former case, he effectively decides between LEAVE and REMAIN, whereas in the latter case, he effectively decides between REMAIN (by voting for LEAVE) and REMAIN FOREVER (by voting for REMAIN).

Intuitively, an agent's voting incentives should depend on the probability with which he believes to be pivotal for LEAVE reaching the κ-majority conditional on being pivotal for either LEAVE or REMAIN reaching the κ-majority. Reasonable equilibrium beliefs should in turn depend on how an agent expects the other agents to vote and thus on the probability with that each other agent is a LEAVE-winner, p_t. In Appendix B, we apply the consistency notion from Kreps and Wilson (1982) to the version of our model with a large finite electorate in order to motivate reasonable beliefs in our model with a continuum electorate. Intuitively, this notion implies that if an agent expects that, for given p_t, each other agent is more likely to vote for LEAVE (REMAIN), then, conditional on being pivotal, he believes with probability 1 (with probability 0) to be pivotal for whether LEAVE reaches the κ-majority.

The current mass of LEAVE-winners p_t is payoff-relevant under two-sided majority voting. Let $\lambda(\pi_t^i, p_t)$ denote the probability with which an agent of type $\pi_t^i \in \{0, 1\}$ votes for LEAVE given p_t. A Markov strategy is then described by $(\lambda(0, p_t), \lambda(1, p_t))$. Let $\mu(p_t)$ describe the probability with which a voter believes to be pivotal for LEAVE reaching the κ-majority when

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supermajority. If no candidate is elected in an initial phase, there are runoff elections between the candidates who received the most votes until one of them obtains a two-thirds majority.

[15]In particular, our analysis extends to the case where LEAVE requires a simple majority and REMAIN becomes permanent if it reaches a certain supermajority.

the current mass of LEAVE-winners is p_t (conditional on being pivotal). The belief system $\mu(p_t)$ is *consistent* with the Markov strategy $(\lambda(0, p_t), \lambda(1, p_t))$ if, for all p_t, the following conditions hold:

$$\begin{cases} \mu(p_t) = 1 & \text{if } l_t > \frac{1}{2} \\ \mu(p_t) \in [0, 1] & \text{if } l_t = \frac{1}{2} \quad \text{with } l_t = p_t \lambda(1, p_t) + (1 - p_t)\lambda(0, p_t). \\ \mu(p_t) = 0 & \text{if } l_t < \frac{1}{2} \end{cases} \tag{8}$$

We employ the following extended equilibrium notion: The Markov strategy $(\lambda(0, p_t), \lambda(1, p_t))$ together with the belief system $\mu(p_t)$ forms an equilibrium if (i) given the belief system, the Markov strategy is optimal under pivotal voting for the continuation values it generates, and (ii) the belief system is consistent with the Markov strategy.[16]

Equilibrium Analysis. In contrast to majority voting, an equilibrium must now implement the optimal policy if it is optimal to never LEAVE, and if the future is sufficiently important:

Proposition 3. *Let $\delta^{TM} \equiv (1 + F(1/2)(1 - 2\bar{p}))^{-1}$. Suppose that the environment is REMAIN-friendly which implies $\delta^{TM} \in (\delta^*, \min\{\delta^M, 1\})$ and let $\delta > \delta^{TM}$. Then, for all κ, two-sided κ-majority voting uniquely implements the optimal policy.*

An intuition for the qualitative difference to (one-sided) κ-majority voting is as follows: Assume, by contradiction, that there exists an equilibrium of two-sided κ-majority voting where LEAVE is chosen with positive probability. For all p_t where LEAVE is chosen, belief consistency implies that each agent believes to be pivotal for LEAVE reaching the κ-majority. An agent effectively decides then between LEAVE and REMAIN, i.e., he faces a trade-off that is like that under (one-sided) κ-majority voting: LEAVE-losers have a strict incentive to vote for REMAIN, but LEAVE-winners can have an incentive to vote LEAVE if they expect the future consequences of REMAIN and LEAVE to be similar.

However, since LEAVE-losers vote REMAIN for any p_t and for any belief, the 50%-majority will be missed at least in periods where a majority of agents are LEAVE-losers. This is where

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[16]The extended equilibrium notion reduces to our original equilibrium notion for majority voting. The belief system is trivial for this mechanism since there is always only a single event in which the agent is pivotal.
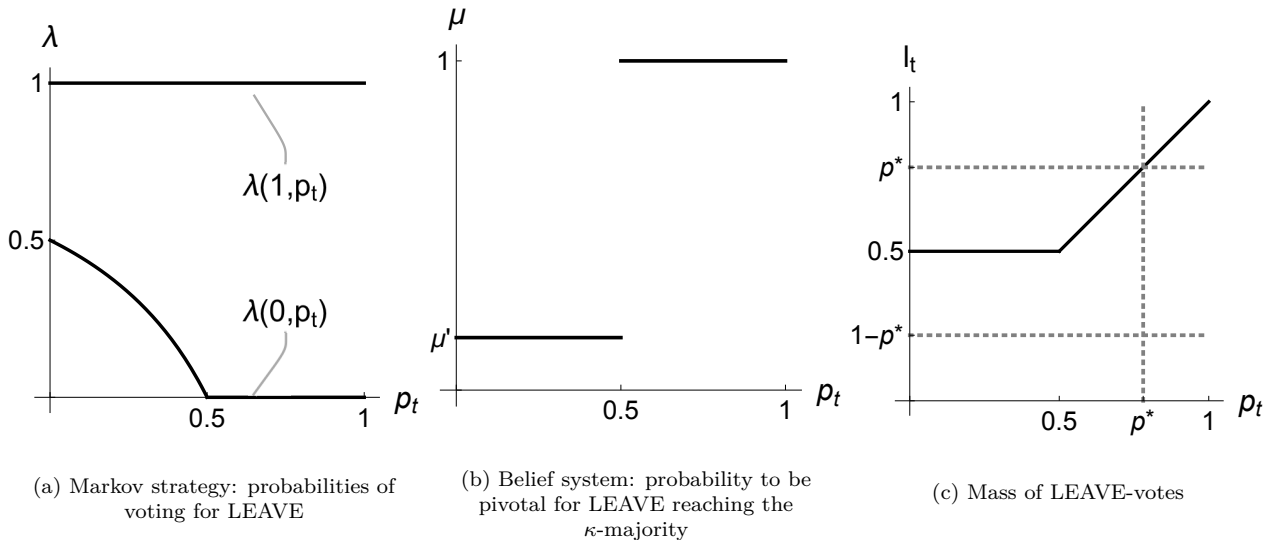
(a) Markov strategy: probabilities of voting for LEAVE

(b) Belief system: probability to be pivotal for LEAVE reaching the κ-majority

(c) Mass of LEAVE-votes

Figure III: The equilibrium behind Proposition 4 $[\gamma = \frac{3}{4}, \delta = 0.77, \kappa = p^*]$

the contrast to normal majority voting kicks in. For such periods, consistency of beliefs implies that each agent believes to be pivotal for REMAIN, or for REMAIN FOREVER. As all agents prefer REMAIN FOREVER over REMAIN if they believe that REMAIN leads to suboptimal decisions in the future, REMAIN eventually leads with a probability of at least $F(\frac{1}{2})$ to REMAIN FOREVER. This drives a wedge between individual assessments of the future consequences of REMAIN and LEAVE. If the future is sufficiently important, this wedge is large enough, and LEAVE-winners always prefer REMAIN over LEAVE, contradicting the choice of LEAVE in some periods.

If it is socially optimal to sometimes LEAVE, the optimal policy can be implemented by one-sided and by two-sided supermajority voting, but doing so requires fine-tuning. The same supermajority rule is optimal in both cases:

Proposition 4. *Let δ^* as introduced in (2): $\delta^* = \frac{1}{2(1-\bar{p})}$. If $\delta < \delta^*$, two-sided p^*-majority voting implements the optimal policy.*

Crucial for the implementation of the optimal policy is that REMAIN FOREVER is never chosen on the equilibrium path. An intuition for how this property arises is as follows: Under the assumption that future decisions are socially optimal, all agents prefer REMAIN over REMAIN FOREVER, but only LEAVE-winners prefer LEAVE over REMAIN. Each LEAVE-winner always votes for LEAVE, whereas the voting incentives of a LEAVE-loser depend on the

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belief $\mu(p_t)$. If $p_t < \frac{1}{2}$ and if the voting behavior of LEAVE-losers is such that $l_t = \frac{1}{2}$, any belief is consistent with the voting behavior. Since a LEAVE-loser prefers REMAIN over REMAIN FOREVER, and prefers REMAIN over LEAVE, there exists an intermediate belief $\mu' \in (0, 1)$ under which such an agent is indeed indifferent between voting for REMAIN and for LEAVE, rendering mixing optimal. Conversely, mixing with the right probability leads to $l_t = \frac{1}{2}$. Since this means that REMAIN always misses the κ-majority, we obtain the desired property. If $p_t > \frac{1}{2}$, a LEAVE-loser believes to be pivotal for LEAVE reaching the κ-majority. Voting for REMAIN is optimal for the same reasons as under normal κ-majority voting. This means that LEAVE obtains the κ-majority only if $p_t \geq \kappa$ and that the optimal policy is implemented for $\kappa = p^*$. Figure III illustrates the equilibrium behind Proposition 4.

Two-sided p^*-majority voting does not uniquely implement the optimal policy under the assumptions of Proposition 4. There exist additional equilibria where agents are indifferent between REMAIN and REMAIN FOREVER. In such an equilibrium, an agent believes that he is never pivotal for LEAVE reaching the κ-majority (conditional on being pivotal), and LEAVE is never chosen on the equilibrium path. Yet the multiplicity of equilibria in this context does not pose a real economic problem. If we consider the version of the model with a large finite electorate, and if we introduce an arbitrarily small, exogenous probability with which an agent votes myopically, equilibria where LEAVE is never chosen cease to exist.

In environments where it is sometimes optimal to LEAVE, fine-tuning matters and the role of the supermajority rule is exactly as under (one-sided) majority voting if agents play the responsive equilibrium that is behind Proposition 4. An important difference is that, in environments where it is socially optimal to REMAIN FOREVER and where the future is sufficiently important, the socially optimal policy is uniquely implemented without fine-tuning (Proposition 3). As a consequence, the large asymmetry in the potential welfare costs from non-optimal rules that we observed for (one-sided) majority voting does not occur under two-sided majority voting. The majority requirement is less important.

6. Conclusion

We studied two voting games where aggregate opinions fluctuate over time. In the first game, one alternative (LEAVE) is irreversible whereas the other (REMAIN) is reversible. In the second, REMAIN also becomes permanent as soon as it wins by a sufficiently large margin. In applications, the potential permanence of REMAIN may be only implicit. While we framed our analysis in terms of Brexit, it applies to other collective decision problems, such as long-term infrastructure projects where preferences may change over time, e.g., the decision to exit nuclear energy, to build a dam, or to develop land.

Our analysis suggests two roles for supermajority voting rules in the game where only LEAVE is permanent: such rules reflect the option value that comes with REMAIN when agents vote myopically and, in environments where LEAVE is likely to have devastating welfare consequences, they can prevent myopic voting behavior. We also find that a too small supermajority can have a much higher welfare cost than a too large supermajority.

In the game where both decisions can become permanent, the second role of supermajority rules disappears and choosing a rather high supermajority to be "on the safe side" becomes less important. Institutional innovations that transform the first game into the second game can be useful. In particular, *explicitly* using two-sided majority voting can make sense in environments where an *implicit* commitment to REMAIN does not automatically occur when REMAIN wins by a large margin. In addition, two-sided supermajority voting may be politically easier to implement because it treats the alternatives more symmetrically. On the downside, it requires that a commitment to REMAIN is in principle possible.

Choices among more alternatives would call for new kinds of mechanisms: for example higher majorities may be necessary for more extreme options, or decisions may be made sequentially (e.g. about whether and how to exit).[17] Another interesting extension would be to embed the problem into a framework where elected political parties negotiate the terms and implement the decision (this is similar to Alesina and Tabellini (1990) and Grüner (2017) who look at

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[17]See Erlenmaier and Gersbach (2001) and Gershkov et al. (2017) for the analysis of related mechanisms in static frameworks.

government spending). Finally, one could also introduce learning about various aspects of the environment.[18]

Appendix A. Extensions

Extension I: Less Polarization

We assumed in our base model that an agent's stage payoff from LEAVE can take only two values, $\pi_t^i = 1$ and $\pi_t^i = 0$. The electorate was polarized in the sense that the median agent's payoff was 1 for $p_t > \frac{1}{2}$ and 0 for $p_t < \frac{1}{2}$.

We now relax the assumption of a completely polarized electorate in two steps: Firstly, the mass of LEAVE-winners is now distributed on the interval $[\alpha, 1 - \alpha]$ with $\alpha \in (0, \frac{1}{2})$. Secondly, we introduce unpolarized agents with intermediate stage payoffs $\pi_t^i \in [0, 1]$ such that the average stage payoff from LEAVE across agents, p_t, does not change. This is done as follows: suppose that, conditional on p_t, π_t^i is 1 with probability $p_t - \alpha$, 0 with probability $1 - p_t - \alpha$, and it is drawn with probability 2α according to a c.d.f G with a p.d.f g that is symmetric around $\frac{1}{2}$ and has support $[0, 1]$. Everything else stays as in our base model. The electorate is completely polarized for $\alpha = 0$ and it converges to an unpolarized electorate as $\alpha \to \frac{1}{2}$.

Consider κ-majority voting. A $(1 - \kappa)$-fraction of agents can enforce REMAIN. That is, the decision is effectively taken by the $(1 - \kappa)$-quantile agent. Because of our continuum electorate assumption, this agent's stage payoff from LEAVE is a deterministic function of p_t. We denote the $(1 - \kappa)$-quantile agent's stage payoff by $\pi^{(1-\kappa)}(p_t)$. See Appendix B for a derivation of the functional form of $\pi^{(1-\kappa)}$. The equilibrium behavior follows from comparing the $(1-\kappa)$-quantile agent's advantage of LEAVE in the current period, $\pi^{(1-\kappa)}(p_t) - \frac{1}{2}$, with the future advantage of REMAIN.

Only cutoff policies can be induced.[19] Each agent's future advantage of REMAIN from cutoff policy p is still $\Delta(p)$ as defined in equation (4). It implies now that $\Delta(p)$ is piecewise constant on $(-\infty, \alpha]$ and on $[1 - \alpha, \infty)$. An interior cutoff $p \in (\alpha, 1 - \alpha)$ is implementable if

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[18]See, for example, Strulovici (2010) and Chan et al. (2018) for the effects of learning about the environment in dynamic, collective decision problems.

[19]This follows from the fact that, as in our base model, $\pi^{(1-\kappa)}(p_t)$ is non-decreasing in p_t, and continuation values do not depend on p_t as Markov strategies condition only π_t^i.

$\pi^{(1-\kappa)}(p) - \frac{1}{2} = \Delta(p)$. For the implementability of non-interior cutoffs we obtain conditions that are analogous to those in our base model. Since $\pi^{(1-\kappa)}(p_t)$ is weakly decreasing in κ, the majority rule κ serves as an instrument for increasing the induced cutoff by making the pivotal agent less eager to LEAVE. This allows us to give a graphical intuition for equilibrium behavior and for the relations to our base model.

Example 1: Unpolarized agents with uniformly distributed stage payoffs. Suppose that $g(\pi_t^i) = 1$. The black curves in Figure IV display $\Delta(p)$ in a REMAIN-friendly environment for various discount factors.[20] All cutoff policies p such that $\Delta(p)$ falls into the gray correspondence in the left panel (the right panel) are implementable by the simple majority rule (by a supermajority rule with $\kappa \approx 0.59$). The optimal policy p^* is still determined by the unique intersection of $p - \frac{1}{2}$ and $\Delta(p)$.[21] The black dot in each panel indicates (p^*, Δ^*).[22]

Suppose first that δ is sufficiently small such that $\Delta(p) \in [0, \frac{1}{2})$ for all p (Figure IVa). An interior cutoff $p^* \in (\frac{1}{2}, 1-\alpha)$ is then optimal. Since $\pi^{(1/2)}(p) - \frac{1}{2} > p - \frac{1}{2}$ for all $p \in (\frac{1}{2}, 1-\alpha]$, the simple majority rule uniquely implements a cutoff that is too small from a social perspective (see the left panel), whereas the right supermajority rule uniquely implements the optimal policy (see the right panel).

Suppose next that δ is large enough such that $\Delta(1 - \alpha) > \frac{1}{2}$ but small enough such that the simple majority rule gives rise to multiple equilibria (Figure IVb). It is then possible to uniquely implement the optimal policy by choosing a sufficiently large supermajority rule (as displayed in the right panel).

Finally, for very large δ (see Figure IVc) it can happen that the majority rule does not matter. The simple majority rule and any supermajority rule uniquely implement then the optimal policy.
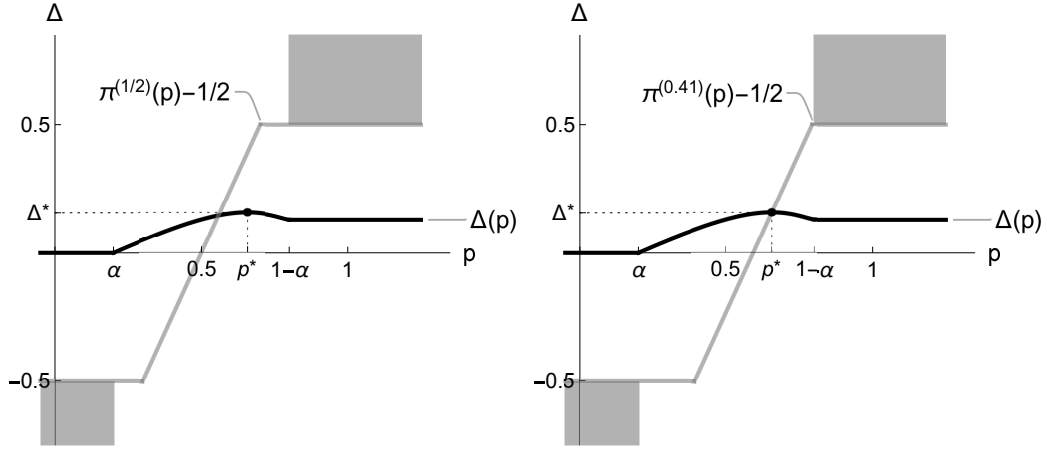
Qualitatively, the three cases and the role supermajority rules play in these cases are analogous to the three cases in Proposition 1.
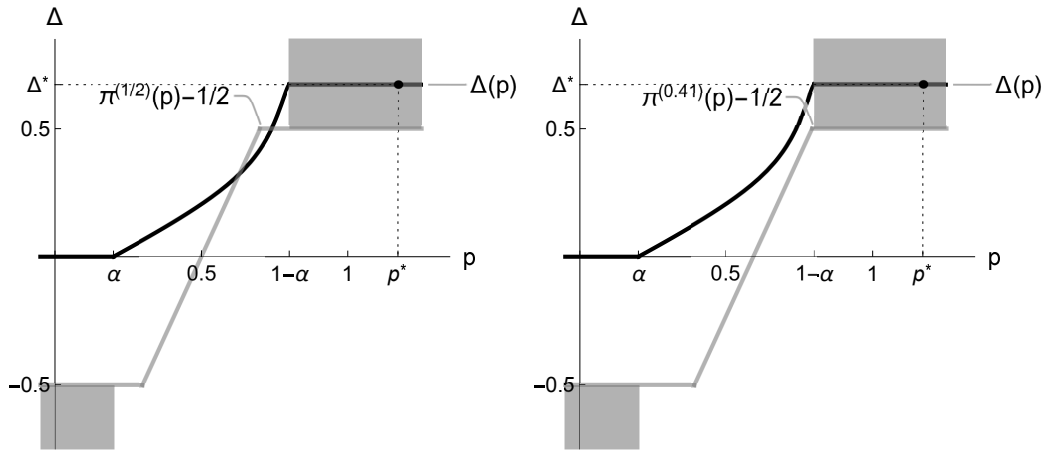

(a) $\kappa = \frac{1}{2}$ (left panel) and $\kappa = \kappa^* \approx 0.59$ (right panel) for $\delta = 0.75$

(b) $\kappa = \frac{1}{2}$ (left panel) and $\kappa \approx 0.59$ (right panel) for $\delta = 0.94$
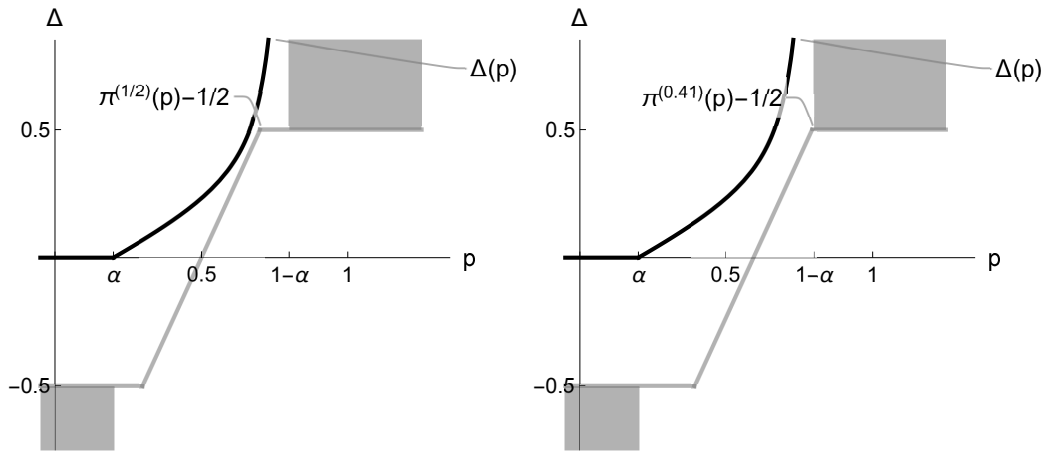
(c) $\kappa = \frac{1}{2}$ (left panel) and $\kappa \approx 0.59$ (right panel) for $\delta = 0.99$

Figure IV: Implementability conditions under κ-majority voting in the model with less polarization
$$[\alpha = \tfrac{1}{5}, F(p_t) = F_{3/4}(\tfrac{p_t - \alpha}{1 - 2\alpha}), g(\pi_t^i) = 1]$$

Implementable polices: Policies where the gray correspondence and the black curve intersects.
All cutoffs $p > 1 - \alpha$, and all cutoffs $p \le \alpha$ describe the same policy, respectively.

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In Example 1, for all $p_t > \frac{1}{2}$, the median agent is more eager to LEAVE than the planner: he assigns to all $p_t \in (\frac{1}{2}, 1 - \alpha]$ a higher current advantage of LEAVE; i.e.,

$$\pi^{(1/2)}(p_t) - \tfrac{1}{2} > p_t - \tfrac{1}{2}. \tag{A.1}$$

In our base model, this condition was implied by our extreme notion of polarization: The median agent had a stage payoff of 1 whenever the mass of LEAVE-winners exceeded $\frac{1}{2}$. Any weaker notion of polarization that implies (A.1) for $p_t \in (\frac{1}{2}, 1 - \alpha]$ is sufficient for obtaining the general optimality of a supermajority rule. Part (a) of the subsequent Lemma identifies such a notion.

Lemma 3. *(a) Let g be quasi-convex. Then, for all degrees of polarization and for all $p_t \in (\frac{1}{2}, 1 - \alpha]$, the median agent is more eager to LEAVE than the planner.*

(b) Let g be strictly quasi-concave. Then, if the degree of polarization is sufficiently low, there exist $p_t \in (\frac{1}{2}, 1 - \alpha]$ such that the planner is more eager to LEAVE than the median agent.

In Example 1, we discussed a boundary case: g was linear (i.e., it was quasi-convex and quasi-concave, but not strictly quasi-concave). The subsequent example illustrates how our results are affected if g is strictly quasi-concave.

Example 2: Unpolarized agents with stage payoffs that are distributed according to a symmetric triangular distribution. Suppose that $g(\pi_t^i) = \min\{4\pi_t^i, 4(1 - \pi_t^i)\}$. Figure V shows how the median agent's current advantage of LEAVE (solid curves) compares to that of the planner (dotted curves) for four degrees of polarization in the electorate.

Figure Va shows that with 20% polarized agents the planner is always more eager to LEAVE than the median agent. As a consequence, the simple majority rule is optimal as a corner solution. With 40% polarized agents (Figure Vb), the planner is more eager to LEAVE for small p_t, but the median agent is more eager to LEAVE for large p_t.
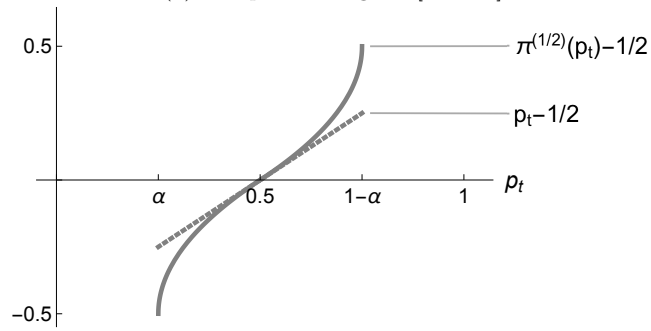
Finally, with 50% or more polarized agents, the median agent is, for all p_t, more eager to LEAVE than the planner (Figures Vc and Vd). The median agent's stage payoff from LEAVE behaves in all important respects like those in Example 1. This becomes particularly apparent
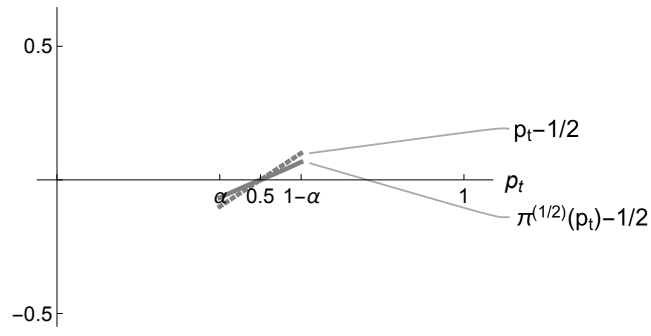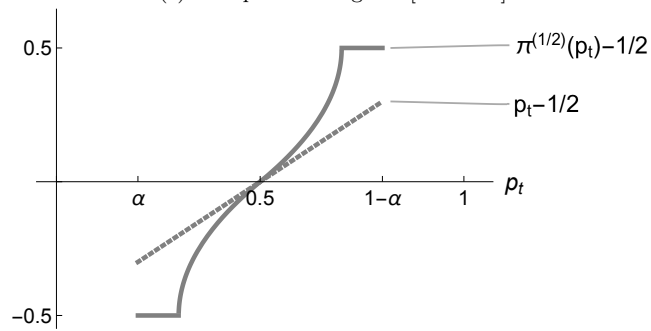
(a) 20% polarized agents [$\alpha = 0.4$]
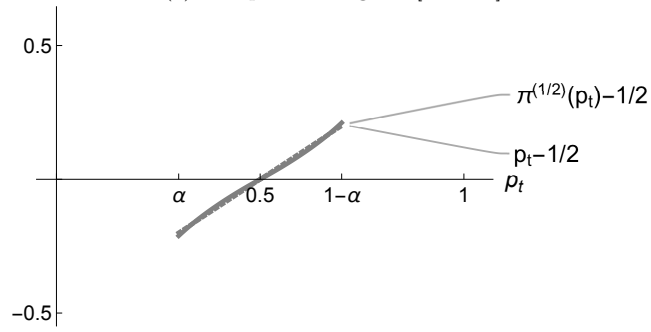
(b) 40% polarized agents [$\alpha = 0.3$]

(c) 50% polarized agents [$\alpha = 0.25$]

(d) 60% polarized agents [$\alpha = 0.2$]

Figure V: Advantage of LEAVE in the current period of the median agent and of the planner
$[g(\pi_t^i) = \min\{4\pi_t^i, 4(1 - \pi_t^i)\}]$

by comparing the solid gray curve in Figure Vd with that in the left panels of Figure IV. The same reasoning as in our discussion of Example 1 applies. Specifically, a supermajority rule is generally optimal, and equilibrium multiplicity can occur for the simple majority rule, but can be avoided by choosing a sufficiently large supermajority rule.

Extension II: Inter-Temporal Correlations

In reality, today's LEAVE-winners are more likely than today's LEAVE-losers to be tomorrow's LEAVE-winners. To study voting in such scenarios, we introduce serial correlation of individual stage payoffs in a simple way that does not affect how the mass of LEAVE-winners fluctuates over time and thus leaves the optimal policy unchanged.

Let $q_1(p_{t-1}, p_t)$ $(q_0(p_{t-1}, p_t))$ denote the probability with which a period $t-1$ LEAVE-winner (LEAVE-loser) becomes a LEAVE-winner in the period t, conditional on p_{t-1} and p_t. We impose the following two assumptions:

Assumption 1. $p_{t-1}q_1(p_{t-1}, p_t) + (1 - p_{t-1})q_0(p_{t-1}, p_t) = p_t$.

Assumption 2. $q_1(p_{t-1}, p_t) \geq q_0(p_{t-1}, p_t)$.

Assumption 1 implies that the mass of LEAVE-winners in period t is still described by p_t. Assumption 2 introduces our notion of positive serial correlation.

Consider κ-majority voting. Since the current mass of LEAVE-winners affects the updated probability to be a LEAVE-winner in the next period, p_t is in this version of the model payoff-relevant. If all agents vote according to the Markov strategy $\lambda(\pi_t^i, p_t)$, the decision is

$$d(p_t) = \begin{cases} L & \text{if } p_t\lambda(1, p_t) + (1 - p_t)\lambda(0, p_t) \geq \kappa \\ R & \text{if } p_t\lambda(1, p_t) + (1 - p_t)\lambda(0, p_t) < \kappa \end{cases}. \tag{A.2}$$

Consider the system of linear equations

$$\begin{cases} V_L(\pi_{t-1}^i, p_{t-1}) = \mathbb{E}[\pi_t^i + \delta V_L(\pi_t^i, p_t)|\pi_{t-1}^i, p_{t-1}] \\ V_R(\pi_{t-1}^i, p_{t-1}) = \mathbb{E}[\mathbf{1}_{d(p_t)=L}(\pi_t^i + \delta V_L(\pi_t^i, p_t)) + \mathbf{1}_{d(p_t)=R}(\frac{1}{2} + \delta V_R(\pi_t^i, p_t))|\pi_{t-1}^i, p_{t-1}] \end{cases} \tag{A.3}$$

implied by policy (A.2). It possesses a unique solution. $\delta V_d(\pi_t^i, p_t)$ describes the continuation value of an agent of type π_t^i from decision $d \in \{R, L\}$. The continuation values affect voting

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incentives only through the future advantage of REMAIN,

$$\Delta(\pi_t^i, p_t) \equiv \delta(V_R(\pi_t^i, p_t) - V_L(\pi_t^i, p_t)).$$

A Markov strategy is optimal under pivotal voting if and only if it satisfies a condition that is analogous to (7).

The future advantage of REMAIN is now a more complicated object than its analog in the base model. Yet, the structure imposed by Assumptions 1 and 2 implies that there is a clear relation between the values that this advantage assumes in both versions of the model:

Lemma 4. *Fix any policy $d(p_t)$ and let $\Delta(\pi_t^i, p_t)$ be the future advantage of REMAIN generated by $d(p_t)$. Then,*

$$\Delta(1, p_t) \leq \Delta \leq \Delta(0, p_t)$$

where Δ is the future advantage of REMAIN implied by $d(p_t)$ in our base model.

Intuitively, the positive serial correlation reinforces incentives: Holding any expectation about future decisions fixed, LEAVE-winners get more eager to vote for LEAVE, while LEAVE-losers get more eager to vote for REMAIN. This has two immediate implications for the implementation and the implementability of cutoff policies:[23]

1) Whenever myopic voting is the unique equilibrium in our base model, it is the unique equilibrium that implements a cutoff policy in the modified model. Put differently, in every scenario where it is optimal to LEAVE under certain conditions, supermajority rules have the same role as in our base model.

2) In scenarios where it is socially optimal to REMAIN forever, supermajority rules can play an even bigger role. In our base model, the optimal policy was generally implementable by the simple majority rule, but not necessarily uniquely implementable. In the equilibrium that implements the optimal policy, all agents vote REMAIN irrespective of their short-term incentives. Supermajority rules were only useful for improving upon the welfare-inferior equi-

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[23]Because of the dependence of Markov strategies on p_t, it is not immediately obvious that only cutoff policies are implementable in our modified model. The analysis of our base model and Lemma 4 allows us thus only to make statements about the implementation and implementability of cutoff policies.

librium or for circumventing its existence. In the extended model, the stronger incentive of LEAVE-winners to vote for LEAVE can render myopic voting the unique equilibrium that implements a cutoff policy.[24] The cutoff policy implemented by any supermajority rule improves then welfare. Interestingly, by a reasoning like in Corollary 1, myopic voting can constitute the **unique** equilibrium that implements a cutoff policy even if all agents agree that it would be better to REMAIN forever, and even if they perceive the long-term consequences of the decision as arbitrarily more important than its short-term consequences.

Appendix B. Proofs

Proofs for Section 3

Proof of Lemma 1

Equation (1) is equivalent to $\phi(p^*, \delta) = 0$, where the auxiliary function $\phi : \mathbb{R} \times (0,1) \to \mathbb{R}$ is defined by

$$\phi(p, \delta) \equiv p - \tfrac{1}{2} - \delta \int_0^p (p - p_t) \mathrm{d}F(p_t). \tag{B.1}$$

For fixed δ, p^* is the unique root of $\phi(p, \delta)$. Two properties of $\phi(p, \delta)$ are important. Firstly, since $\phi_p(p, \delta) = 1 - \delta F(p) > 0$, we obtain for fixed δ that $\phi(p, \delta) < 0$ implies $p^* > p$ and that $\phi(p, \delta) > 0$ implies $p^* < p$. Secondly, p^* is strictly increasing in δ since $\phi(0, \delta) = -\tfrac{1}{2}$ for all δ, and since $\phi_p(p, \delta) > 0$ and $\phi_{p\delta}(p, \delta) < 0$ for $p > 0$.

Because

$$\phi(\tfrac{1}{2}, \delta) = -\delta \int_0^{1/2} (\tfrac{1}{2} - p_t) \mathrm{d}F(p_t) < 0,$$

we obtain $p^* > \tfrac{1}{2}$. Since $\phi(1, \delta) = 1 - \tfrac{1}{2} - \delta(1 - \bar{p})$, we obtain that $\phi(1, \delta) > 0$ for $\bar{p} \geq \tfrac{1}{2}$. Hence, by the Intermediate Value Theorem, $p^* < 1$ for any LEAVE-friendly or neutral environment. If the environment is REMAIN-friendly, the sign of $\phi(1, \delta)$ depends on the discount factor. Since $\delta = \frac{1}{2(1 - \bar{p})}$ solves $\phi(1, \delta) = 0$, we obtain the wished result.

---

[24]An example can be constructed in the following way: Fix a REMAIN-friendly environment. For $\delta = \delta^* + \epsilon$ with $\epsilon > 0$, Δ is larger than $\tfrac{1}{2}$ and converges to $\tfrac{1}{2}$ as $\epsilon \to 0$. By choosing the function q_1 that determines the serial correlation such that $\Delta(1, p_t)$ is bounded away from Δ as $\epsilon \to 0$, we obtain the result.

Proofs for Section 4

Proof of Lemma 2

Suppose that $(\lambda(0), \lambda(1))$ constitutes an equilibrium under κ-majority voting. By the conditions in (7), $\lambda(1) \geq \lambda(0)$. Since this implies that $p_t \lambda(1) + (1 - p_t)\lambda(0)$ is non-decreasing in p_t, (5) defines a cutoff policy.

Suppose $d(p_t)$ is a cutoff policy with cutoff p. By using the structure of this policy and $\mathbb{E}[\pi_t^i | p_t] = p_t$ in system (6), system (6) simplifies to the system (3). Hence, $\Delta = \Delta(p)$.

Auxiliary Results on the Shape of $\Delta(p)$

We next establish the main properties of $\Delta(p)$.

Lemma B.1. (a) $\Delta(p) \geq \Delta(p')$ if and only if the planner weakly prefers cutoff p over cutoff p'.

(b) $\Delta(p)$ is piecewise constant on $(-\infty, 0]$ and on $[1, \infty)$. On the interval $[0, 1]$, $\Delta(p)$ is single-peaked with peak at p^* if $p^* < 1$, and strictly increasing if $p^* \geq 1$.

(c) $\Delta(p^*) = p^* - \frac{1}{2}$. Moreover, $\Delta(p) > p - \frac{1}{2}$ if $p < p^*$ and $\Delta(p) < p - \frac{1}{2}$ if $p > p^*$.

Proof. (a) The planner's ex-ante expected payoff from cutoff policy p is $V_R(p)$. Since the function V_L does not depend on p, $\Delta(p)$ is a positive linear transformation of $V_R(p)$.

(b) It is obvious from condition (4) that Δ is continuous on $\mathbb{R}$ and piecewise constant on $(-\infty, 0]$ and on $[1, \infty)$. For all $p \in (0, 1)$, we have

$$
\begin{aligned}
\Delta'(p) &= \frac{\delta^2 f(p)}{(1 - \delta F(p))^2} \int_0^p (\tfrac{1}{2} - p_t) \mathrm{d}F(p_t) + \frac{\delta}{1 - \delta F(p)}(\tfrac{1}{2} - p)f(p) \\
&= -\frac{\delta f(p)}{(1 - \delta F(p))^2}\left(-\delta \int_0^p (\tfrac{1}{2} - p_t)\mathrm{d}F(p_t) - (1 - \delta F(p))(\tfrac{1}{2} - p) \right) \\
&= -\frac{\delta f(p)}{(1 - \delta F(p))^2}\left(p - \tfrac{1}{2} - \delta \int_0^p (p - p_t)\mathrm{d}F(p_t) \right) \\
&\overset{(B.1)}{=} -\frac{\delta f(p)}{(1 - \delta F(p))^2}\phi(p, \delta).
\end{aligned}
$$

By the reasoning in the proof of Lemma 1, for any fixed δ, $\phi(p, \delta)$ is strictly increasing in p with a root at p^*. If $p^* \geq 1$, we obtain $\Delta'(p) > 0$ for all $p \in (0, 1)$. If $p^* < 1$, we obtain $\Delta'(p) > 0$ for $p \in (0, p^*)$, and $\Delta'(p) < 0$ for $p \in (p^*, 1)$.

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(c) We have

$$
\begin{aligned}
\Delta(p) - (p - \tfrac{1}{2}) &\overset{(4)}{=} \frac{1}{1 - \delta F(p)}\left(\delta \int_0^p (\tfrac{1}{2} - p_t)\mathrm{d}F(p_t) - (1 - \delta F(p))(p - \tfrac{1}{2})\right) \\
&= -\frac{1}{1 - \delta F(p)}\left(p - \tfrac{1}{2} - \delta \int_0^p (p - p_t)\mathrm{d}F(p_t)\right) \\
&\overset{(B.1)}{=} -\frac{1}{1 - \delta F(p)}\phi(p, \delta).
\end{aligned}
$$

We obtain $\Delta(p) > p - \tfrac{1}{2}$ for $p < p^*$, $\Delta(p^*) = p^* - \tfrac{1}{2}$, and $\Delta(p) < p - \tfrac{1}{2}$ for $p > p^*$. $\qquad\square$

Lemma B.2. *Fix any REMAIN-friendly environment.*

(a) *Assume that $\delta < \delta^*$. Then $\Delta(p) \in [0, \tfrac{1}{2})$ for all p.*

(b) *Assume that $\delta > \delta^*$. Then there exists a unique value $\Delta^{-1}(\tfrac{1}{2}) \in [\tfrac{1}{2}, 1)$ for $\delta \in (\delta^*, \delta^M]$ and a unique value $\Delta^{-1}(\tfrac{1}{2}) \in (0, \tfrac{1}{2})$ for $\delta > \delta^M$. Moreover, $\Delta(p) \in [0, \tfrac{1}{2})$ for $p < \Delta^{-1}(\tfrac{1}{2})$, and $\Delta(p) > \tfrac{1}{2}$ for $p > \Delta^{-1}(\tfrac{1}{2})$.*

Proof. We divide the argument in three steps. (a) follows from Steps 1 and 2 and (b) from Steps 1 and 3.

Step 1. By Lemma B.1 (b), $\min_p \Delta(p) = \min\{\Delta(0), \Delta(1)\}$. Since $\Delta(0) = 0$, $\Delta(1) = \frac{\delta}{1-\delta}(\tfrac{1}{2} - \bar{p})$, and $\bar{p} < \tfrac{1}{2}$ we obtain that $\min_p \Delta(p) = 0$ in any REMAIN-friendly environment.

Step 2. Suppose that $\delta < \delta^*$. By Lemma B.1 (a), $\Delta(p) \leq \Delta(p^*)$. Since by Lemma B.1 (c), $\Delta(p^*) = p^* - \tfrac{1}{2}$, and by Lemma 1, $p^* < 1$, $\Delta(p) < \tfrac{1}{2}$.

Step 3. Suppose that $\delta > \delta^*$. Then, $p^* > 1$ by Lemma 1. By Lemma B.1 (b) and (c), this implies that $\Delta(p^*) > \tfrac{1}{2}$, and that $\Delta(p)$ is strictly increasing on $[0, 1]$, with $\Delta(1) = \Delta(p^*)$. Since $\Delta(0) = 0$ and since $\Delta(p)$ is continuous, we can apply the Intermediate Value Theorem to obtain a unique value $\Delta^{-1}(\tfrac{1}{2}) \in (0, 1)$. Since $\Delta(p)$ is strictly increasing on $[0, 1]$ and weakly increasing on $\mathbb{R}$, $\Delta(p) < \tfrac{1}{2}$ for $p < \Delta^{-1}(\tfrac{1}{2})$ and $\Delta(p) > \tfrac{1}{2}$ for $p > \Delta^{-1}(\tfrac{1}{2})$. Finally,

$$
\begin{aligned}
\Delta^{-1}(\tfrac{1}{2}) < \tfrac{1}{2} \Leftrightarrow \Delta(\tfrac{1}{2}) > \tfrac{1}{2} \;\Leftrightarrow\; & \frac{\delta}{1 - \delta F(\tfrac{1}{2})} \int_0^{1/2} (\tfrac{1}{2} - p_t)\mathrm{d}F(p_t) > \tfrac{1}{2} \\
\Leftrightarrow\; & \delta 2 \int_0^{1/2} (\tfrac{1}{2} - p_t)\mathrm{d}F(p_t) > 1 - \delta F(\tfrac{1}{2}) \\
\Leftrightarrow\; & \delta 2 \int_0^{1/2} (1 - p_t)\mathrm{d}F(p_t) > 1 \Leftrightarrow \delta > \delta^M. \qquad\square
\end{aligned}
$$

Lemma B.3. *Fix any LEAVE-friendly or neutral environment.*

(a) Assume that $\delta < \frac{1}{2\bar{p}}$. Then $\Delta(p) \in (-\frac{1}{2}, \frac{1}{2})$ for all p.

(b) Assume that $\delta > \frac{1}{2\bar{p}}$. Then there exists a unique value $\Delta^{-1}(-\frac{1}{2}) \in (p^*, 1)$. Moreover, $\Delta(p) \in (-\frac{1}{2}, \frac{1}{2})$ for $p < \Delta^{-1}(-\frac{1}{2})$ and $\Delta(p) < -\frac{1}{2}$ for $p > \Delta^{-1}(-\frac{1}{2})$.

Proof. (a) follows from Steps 1 and 2.1, (b) from Steps 1 and 2.2.

Step 1. By Lemma B.1 (a), $\Delta(p) \leq \Delta(p^*)$. Since $\Delta(p^*) = p^* - \frac{1}{2}$, and since, by Lemma 1, $p^* \in (\frac{1}{2}, 1)$ for all LEAVE-friendly or neutral environments, $\Delta(p) < \frac{1}{2}$.

Step 2. By Lemma B.1 (b), $\min_p \Delta(p) = \min\{\Delta(0), \Delta(1)\}$. Since (4) implies $\Delta(0) = 0$ and $\Delta(1) = \frac{\delta}{1-\delta}(\frac{1}{2} - \bar{p})$, and since $\bar{p} \geq \frac{1}{2}$ in any LEAVE-friendly or neutral environment, we obtain that $\min_p \Delta(p) = \Delta(1)$. Thus,

$$\min_p \Delta(p) - (-\tfrac{1}{2}) = \frac{\delta}{1-\delta}(\tfrac{1}{2} - \bar{p}) - (-\tfrac{1}{2}) = \frac{\bar{p}}{1-\delta}(\frac{1}{2\bar{p}} - \delta). \tag{B.2}$$

Step 2.1. Suppose that $\delta < \frac{1}{2\bar{p}}$. Then, (B.2) implies $\Delta(p) > -\frac{1}{2}$.

Step 2.2. Suppose that $\delta > \frac{1}{2\bar{p}}$. Then, (B.2) and $\min_p \Delta(p) = \Delta(1)$ imply $\Delta(1) < -\frac{1}{2}$. In Step 1, we have already observed that $\Delta(p^*) \in (0, \frac{1}{2})$ with $p^* \in (0, 1)$. Since $\Delta(p)$ is continuous, we can apply the Intermediate Value Theorem to obtain a unique value $\Delta^{-1}(-\frac{1}{2}) \in (p^*, 1)$. Furthermore, it follows from Lemma B.1 (b) that $\Delta(p) > -\frac{1}{2}$ for $p < \Delta^{-1}(-\frac{1}{2})$ and $\Delta(p) < -\frac{1}{2}$ for $p > \Delta^{-1}(-\frac{1}{2})$. $\qquad\square$

Proof of Proposition 1

Note that

$$\delta^* = \frac{1}{2\int_0^1 (1-p_t)\mathrm{d}F(p_t)} < \delta^M = \frac{1}{2\int_0^{1/2}(1-p_t)\mathrm{d}F(p_t)}.$$

Case i: $\delta \in (0, \delta^*)$**.** Suppose that $(\lambda(0), \lambda(1))$ constitutes an equilibrium. By Lemma 2, $\Delta = \Delta(p)$ for some p. Since, by Lemma B.2 (a) $\Delta(p) \in [0, \frac{1}{2})$ for all p, $(\lambda(0), \lambda(1))$ is by (7) optimal for the Δ it implies if and only if $(\lambda(0), \lambda(1)) = (0, 1)$. In other words, myopic voting constitutes the unique equilibrium. Since, by (5), myopic voting induces cutoff policy κ, we obtain the result.

Case ii and iii: $\delta \in (\delta^*, 1)$**.** The subsequent Steps 1–3 do not differentiate between Case ii ($\delta \leq \delta^M$) and Case iii ($\delta > \delta^M$). However, by Lemma B.2 (b), the set $[\frac{1}{2}, \Delta^{-1}(\frac{1}{2})]$ is empty in Case iii while $\Delta^{-1}(\frac{1}{2}) \in [\frac{1}{2}, 1)$ in Case ii.

Step 1: For all κ, always voting for REMAIN, $(\lambda(0), \lambda(1)) = (0, 0)$, constitutes an equilibrium. By (5), any cutoff policy $p' > 1$ describes the policy implemented by $(\lambda(0), \lambda(1)) = (0, 0)$. Fix any such p'. By Lemma 2, $\Delta = \Delta(p')$. Since $1 < p'$ and since $\Delta^{-1}(\frac{1}{2}) < 1$ by Lemma B.2 (b), we obtain that $\Delta(p') > \frac{1}{2}$. By (7) this implies that $(\lambda(0), \lambda(1)) = (0, 0)$ is optimal under pivotality considerations for the Δ it induces.

Step 2: If $\kappa \in [\frac{1}{2}, \Delta^{-1}(\frac{1}{2})]$, myopic voting, $(\lambda(0), \lambda(1)) = (0, 1)$, constitutes an equilibrium. By (5), myopic voting induces cutoff policy $p = \kappa$. By Lemma 2, $\Delta = \Delta(\kappa)$. Since $\kappa \leq \Delta^{-1}(\frac{1}{2})$ and Lemma B.2 (b) imply that $\Delta(\kappa) \in [0, \frac{1}{2}]$, we obtain by (7) that myopic voting is optimal under pivotality considerations for the Δ it induces.

Step 3: If $\kappa \in (\Delta^{-1}(\frac{1}{2}), 1)$, the equilibrium from Step 1 is unique. Assume to the contrary that $(\lambda(0), \lambda(1)) \neq (0, 0)$ constitutes a further equilibrium. By Lemma 2, $\Delta = \Delta(p)$ for some p. Since $\Delta(p) \geq 0$ by Lemma B.2 (b), $\lambda(0) = 0$. This implies $p \geq \kappa$ by (5). However, since $\kappa > \Delta^{-1}(\frac{1}{2})$ and Lemma B.2 (b) imply $\Delta(p) > \frac{1}{2}$, pivotality implies also $\lambda(1) = 0$ by (7), yielding a contradiction.

Step 4: The statements about the optimal majority rule. By Lemma 1, it is optimal to never LEAVE for $\delta > \delta^*$. By Step 1, the optimal policy is implementable by κ-majority voting with any κ. By Steps 2 and 3, the optimal policy is uniquely implemented by κ-majority voting only if $\kappa > \Delta^{-1}(\frac{1}{2})$. Recall that this condition is satisfied for all $\kappa \in [\frac{1}{2}, 1)$ if $\delta > \delta^M$.

Proof of Corollary 1

Note that $\delta^M \geq 1$ is equivalent to

$$2 \int_0^{1/2} (1 - p_t) f(p_t) \mathrm{d}p_t \leq 1. \tag{B.3}$$

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Consider REMAIN-friendly power distribution functions F_γ, $\gamma \in (0,1)$. Since $F_\gamma'(p_t) = \frac{1}{\gamma}(1 - p_t)^{\frac{1}{\gamma}-1}$, the left-hand side of (B.3) can be written as

$$2\int_0^{1/2} \frac{1}{\gamma}(1 - p_t)^{\frac{1}{\gamma}}dp_t = 2\left[\frac{-1}{1+\gamma}(1 - p_t)^{\frac{1}{\gamma}+1}\right]_{p_t=0}^{p_t=1/2} = \frac{1}{1+\gamma}\left(2 - \left(\tfrac{1}{2}\right)^{\frac{1}{\gamma}}\right).$$

Since this expression is continuous in γ and converges to $\frac{3}{4}$ as $\gamma \to 1$, we obtain that $\delta^M \geq 1$ for any γ sufficiently close to 1.

Suppose the environment is REMAIN-friendly and that $\delta^M \geq 1$. Then, by Proposition 1, for any $\delta > \delta^*$, the cutoff policy $\frac{1}{2}$ is implementable by the simple majority rule. By the same Proposition, any sufficiently large supermajority rule uniquely implements the optimal policy $p^* > 1$. Hence, the simple majority rule can lead to a welfare loss of

$$V_R(p^*) - V_R(\tfrac{1}{2}) = \tfrac{1}{\delta}\left(\Delta(p^*) - \Delta(\tfrac{1}{2})\right)$$

where the equality follows because $V_L(p^*) = V_L(\frac{1}{2})$ by the irreversibility of LEAVE. For $\delta \leq \delta^M$, Lemma B.2 (b) implies that $\Delta(\frac{1}{2}) \leq \frac{1}{2}$. On the other hand, (4) and $p^* > 1$ imply that $\Delta(p^*) = \frac{\delta}{1-\delta}(\frac{1}{2} - \bar{p})$. Since $\frac{1}{2} - \bar{p} > 0$ in any REMAIN-friendly environment, $\Delta(p^*)$ grows without bound as $\delta \to 1$. Hence, $V_R(p^*) - V_R(\frac{1}{2})$ grows without bound as $\delta \to 1$.

Proof of Proposition 2

Case i: $\delta \in (0, \frac{1}{2\bar{p}})$. Since, by Lemma B.3, this condition implies $\Delta(p) \in (-\frac{1}{2}, \frac{1}{2})$ for all p, a reasoning analogous to that in Case i of Proposition 1 applies.

Case ii: $\delta \in (\frac{1}{2\bar{p}}, 1)$. We know that $\Delta^{-1}(-\frac{1}{2}) \in (p^*, 1)$ from Lemma B.3 (b). We show that for all $\kappa \in [\frac{1}{2}, 1)$ a unique equilibrium exists and that the implemented policy is as asserted in the Proposition.

Step 1: For all $\kappa \in [\frac{1}{2}, \Delta^{-1}(-\frac{1}{2})]$, myopic voting, $(\lambda(0), \lambda(1)) = (0, 1)$, constitutes an equilibrium. By (5), myopic voting induces cutoff policy $p = \kappa$. By Lemma 2, $\Delta = \Delta(\kappa)$. Lemma B.3 (b), $\delta > \frac{1}{2\bar{p}}$ and $\kappa \in [\frac{1}{2}, \Delta^{-1}(-\frac{1}{2})]$ imply $\Delta(\kappa) \in [-\frac{1}{2}, \frac{1}{2})$. Thus, by (7), myopic voting is optimal under pivotality considerations for the Δ it induces.

Step 2: For all $\kappa \in [\frac{1}{2}, \Delta^{-1}(-\frac{1}{2})]$, no other equilibrium exists. Assume to the contrary that $(\lambda(0), \lambda(1)) \neq (0, 1)$ constitutes a further equilibrium. By Lemma 2, $\Delta = \Delta(p)$ for some p.

Since, by Lemma B.3 (b), $\Delta(p) < \frac{1}{2}$, pivotality implies $\lambda(1) = 1$ by (7). Thus, $\lambda(0) > 0$ must be true. By (5), $\lambda(1) = 1$ and $\lambda(0) > 0$ imply that a cutoff $p < \kappa$ is induced. Since $\kappa \leq \Delta^{-1}(-\frac{1}{2})$ by our supposition, $p < \Delta^{-1}(-\frac{1}{2})$. Since this implies for $\delta > \frac{1}{2\bar{p}}$ that $\Delta(p) > -\frac{1}{2}$ by Lemma B.3 (b), only $\lambda(0) = 0$ is optimal under pivotality by (7), yielding a contradiction.

Step 3: For all $\kappa \in (\Delta^{-1}(-\frac{1}{2}), 1)$,

$$(\lambda(0), \lambda(1)) = (\tfrac{\kappa - \Delta^{-1}(-1/2)}{1 - \Delta^{-1}(-1/2)}, 1)$$

constitutes an equilibrium. By (5), the strategy $(\frac{\kappa - \Delta^{-1}(-1/2)}{1 - \Delta^{-1}(-1/2)}, 1)$ induces the cutoff policy $\Delta^{-1}(-\frac{1}{2})$. By Lemma 2, $\Delta = \Delta(\Delta^{-1}(-\frac{1}{2})) = -\frac{1}{2}$. Hence, by (7), $(\frac{\kappa - \Delta^{-1}(-1/2)}{1 - \Delta^{-1}(-1/2)}, 1)$ is optimal under pivotality for the Δ it induces.

Step 4: For all $\kappa \in (\Delta^{-1}(-\frac{1}{2}), 1)$, *no other equilibrium exists.* Assume to the contrary that

$$(\lambda(0), \lambda(1)) \neq (\tfrac{\kappa - \Delta^{-1}(-1/2)}{1 - \Delta^{-1}(-1/2)}, 1)$$

constitutes a further equilibrium. By Lemma 2, $\Delta = \Delta(p)$ for some p. Since $\Delta(p) < \frac{1}{2}$ by Lemma B.3 (b), pivotality considerations imply $\lambda(1) = 1$ by (7). Thus, $\lambda(0) \neq \frac{\kappa - \Delta^{-1}(-1/2)}{1 - \Delta^{-1}(-1/2)}$. We distinguish two cases:

Suppose first that $\lambda(0) < \frac{\kappa - \Delta^{-1}(-1/2)}{1 - \Delta^{-1}(-1/2)}$. Together with $\lambda(1) = 1$ and (5), this implies that a cutoff $p > \Delta^{-1}(-\frac{1}{2})$ is induced. By Lemma B.3 (b), this implies that $\Delta(p) < -\frac{1}{2}$ for $\delta > \frac{1}{2\bar{p}}$. Thus, by (7), only $\lambda(0) = 1$ is optimal under pivotality, yielding a contradiction.

Suppose next that $\lambda(0) > \frac{\kappa - \Delta^{-1}(-1/2)}{1 - \Delta^{-1}(-1/2)}$. Then a cutoff $p < \Delta^{-1}(-\frac{1}{2})$ is induced, and only $\lambda(0) = 0$ is optimal, yielding again a contradiction.

Proof of Corollary 2

Fix any LEAVE-friendly environment and consider $\delta > \frac{1}{2\bar{p}}$. By Proposition 2, a policy is implementable by some majority rule κ if and only if it is a cutoff policy $p \in [\frac{1}{2}, \Delta^{-1}(-\frac{1}{2})]$. By Lemma B.3 (b),

$$\min_{p \in [1/2, \Delta^{-1}(-1/2)]} \Delta(p) = -\tfrac{1}{2} \quad \text{and} \quad \max_{p \in [1/2, \Delta^{-1}(-1/2)]} \Delta(p) < \tfrac{1}{2}.$$

Since the welfare loss from cutoff policy p is

$$V_R(p^*) - V_R(p) = \tfrac{1}{\delta}(\Delta(p^*) - \Delta(p)),$$

the maximal welfare loss is strictly smaller than $\frac{1}{\delta}$, and it is bounded by 1 for $\delta \to 1$.

Proofs for Section 5

Recall that in this section we consider the version of the model where votes can be for REMAIN or for LEAVE, but where three decisions are possible in each period, LEAVE (L), REMAIN (R) and REMAIN forever ($R\infty$).

Sensible Beliefs under Voting by Two-Sided κ-majority voting

In our model with a continuum of agents, each pivotality event occurs with probability 0 and Bayes Law is not applicable. In order to obtain an understanding of what reasonable beliefs are, we consider now the version of our model with a large, finite number of n agents, and we derive how the Bayesian belief behaves in the limit. This will motivate our notion of sensible beliefs in our model with a continuum of agents.

Let $m_n^L \equiv \lceil \kappa n \rceil - 1$ and $m_n^R \equiv \lceil (1-\kappa)n \rceil - 1$. If m_n^L other agents vote for LEAVE, i is pivotal for LEAVE reaching the κ-majority; if m_n^R others vote for LEAVE, he is pivotal for REMAIN reaching the κ-majority. Suppose that agent i believes that each other agent votes for LEAVE with probability $k \in (0,1)$. His Bayesian belief about the pivotality scenario is then $\mu = \mu_n(k)$ where

$$\mu_n(k) \equiv \frac{\binom{n-1}{m_n^L} k^{m_n^L} (1-k)^{n-1-m_n^L}}{\binom{n-1}{m_n^R} k^{m_n^R} (1-k)^{n-1-m_n^R} + \binom{n-1}{m_n^L} k^{m_n^L} (1-k)^{n-1-m_n^L}}. \tag{B.4}$$

For $k \in \{0,1\}$, we employ the consistency notion from Kreps and Wilson (1982): The belief μ_n is **consistent** with the probability $k_n \in [0,1]$ in the finite version of the model if there exists a sequence $(k_{n,\tau})_\tau$ in $(0,1)$ with $\lim_{\tau \to \infty} k_{n,\tau} = k_n$ such that $\lim_{\tau \to \infty} \mu_n(k_{n,\tau}) = \mu_n$. We say that the limit belief μ is **consistent** with the limit probability $k \in [0,1]$ if there exists a sequence of probabilities $(k_n)_n$ in $[0,1]$ with $\lim_{n \to \infty} k_n = k$ and a sequence of beliefs $(\mu_n)_n$ with $\lim_{n \to \infty} \mu_n = \mu$ such that, for all n, μ_n is consistent with k_n in the finite version of the model.

Lemma B.4. *For all $\kappa \in (\frac{1}{2}, 1)$, the limit belief $\mu \in [0, 1]$ is consistent with the limit probability $k \in [0, 1]$ if, and only if,*

$$
\begin{cases}
\mu = 1 & \text{if } k > \frac{1}{2} \\
\mu \in [0, 1] & \text{if } k = \frac{1}{2} \quad . \\
\mu = 0 & \text{if } k < \frac{1}{2}
\end{cases}
$$

Proof. For $k \in (0, 1)$, we can rewrite (B.4) as

$$
\begin{aligned}
\mu_n(k) &= \frac{\frac{\binom{n-1}{m_n^L}}{\binom{n-1}{m_n^R}} \left(\frac{k}{1-k}\right)^{m_n^L - m_n^R}}{1 + \frac{\binom{n-1}{m_n^L}}{\binom{n-1}{m_n^R}} \left(\frac{k}{1-k}\right)^{m_n^L - m_n^R}} \\
&= \frac{\frac{(n-1-m_n^R)!}{m_n^L!} \frac{m_n^R!}{(n-1-m_n^L)!} \left(\left(\frac{k}{1-k}\right)^{(m_n^L - m_n^R)/n}\right)^n}{1 + \frac{(n-1-m_n^R)!}{m_n^L!} \frac{m_n^R!}{(n-1-m_n^L)!} \left(\left(\frac{k}{1-k}\right)^{(m_n^L - m_n^R)/n}\right)^n}. \quad (B.5)
\end{aligned}
$$

Step 1: Only belief $\mu_n = 0$ ($\mu_n = 1$) is consistent with probability $k = 0$ ($k = 1$) in the finite version of the model. Fix any n. Let $(k_{n,\tau})_\tau$ be any sequence in $(0, 1)$ with $\lim_{\tau \to \infty} k_{n,\tau} = 0$ ($\lim_{\tau \to \infty} k_{n,\tau} = 1$). $\lim_{\tau \to \infty} \mu_n(k_{n,\tau}) = 0$ ($\lim_{\tau \to \infty} \mu_n(k_{n,\tau}) = 1$) immediately follows from (B.5).

Step 2: Implications of consistency of limit beliefs for probability sequences $(k_n)_n$ with $k_n = k$ for all n: $k \in (0, \frac{1}{2})$ implies $\lim_{n \to \infty} \mu_n(k) = 0$, $k = \frac{1}{2}$ implies $\lim_{n \to \infty} \mu_n(k) = \frac{1}{2}$, and $k \in (\frac{1}{2}, 1)$ implies $\lim_{n \to \infty} \mu_n(k) = 1$. Fix any $k \in (0, 1)$. We note that $\lim_{n \to \infty} \frac{(n-1-m_n^R)!}{m_n^L!} = 1$, $\lim_{n \to \infty} \frac{m_n^R!}{(n-1-m_n^L)!} = 1$, and $\lim_{n \to \infty} \left(\frac{k}{1-k}\right)^{(m_n^L - m_n^R)/n} = \left(\frac{k}{1-k}\right)^{2\kappa - 1}$. Since $k \in (0, \frac{1}{2})$ implies $\left(\frac{k}{1-k}\right)^{2\kappa - 1} < 1$, we obtain from (B.5) that $\lim_{n \to \infty} \mu_n(k) = 0$. Analogously, since $k \in (\frac{1}{2}, 1)$ implies $\left(\frac{k}{1-k}\right)^{2\kappa - 1} > 1$, we obtain from (B.5) that $\lim_{n \to \infty} \mu_n(k) = 1$. Finally, since $k = \frac{1}{2}$ implies $\left(\frac{k}{1-k}\right)^{m_n^L - m_n^R} = 1$, we obtain from (B.5) that $\lim_{n \to \infty} \mu_n(k) = \frac{1}{2}$.

Step 3: Implications of consistency of limit beliefs for probability sequences $(k_n)_n$.

Step 3.1: For all $k \in [0, \frac{1}{2})$ ($k \in (\frac{1}{2}, 1]$), only the limit belief $\mu = 0$ ($\mu = 1$) is consistent with the limit probability k. Let $(k_n)_n$ be a sequence in $[0, 1]$ with $\lim_{n \to \infty} k_n = k$ and let $(\mu_n)_n$ be a sequence in $[0, 1]$ with $\lim_{n \to \infty} \mu_n = \mu$ such that, for all n, belief μ_n is consistent with probability

k_n in the finite version of the model. It follows from Steps 1 and 2 that $\lim_{n\to\infty} \mu_n(k_n) = 0$ if $k < \frac{1}{2}$ and that $\lim_{n\to\infty} \mu_n(k_n) = 1$ if $k > \frac{1}{2}$.

Step 3.2: Any limit belief $\mu \in (0,1)$ is consistent with the limit probability $k = \frac{1}{2}$. Since $\mu_n(k)$ is strictly increasing and continuous in k with $\lim_{k\to 0} \mu_n(k) = 0$ and $\lim_{k\to 1} \mu_n(k) = 1$, for any $\mu \in (0,1)$, there exists by the Intermediate Value Theorem a unique value $\mu_n^{-1}(\mu)$. Set $k_n = \mu_n^{-1}(\mu)$ and $\mu_n = \mu_n(k_n)$. By construction, $\lim_{n\to\infty} \mu_n = \mu$. Moreover, by Step 2, $\lim_{n\to\infty} k_n = \frac{1}{2}$.

Step 3.3: Any limit belief $\mu \in \{0,1\}$ is consistent with the limit probability $k = \frac{1}{2}$. Consider $k_n = \frac{1}{1+n^{1/(m_n^L - m_n^R)}}$ and $\mu_n = \mu_n(k_n)$. Then, since $\lim_{n\to\infty} n^{1/(m_n^L - m_n^R)} = 1$, $\lim_{n\to\infty} k_n = \frac{1}{2}$. Moreover,

$$
\begin{aligned}
\mu_n \leq \frac{2}{n} \quad &\Leftrightarrow \quad \frac{n-2}{n} \frac{(n-1-m_n^R)!}{m_n^L!} \frac{m_n^R!}{(n-1-m_n^L)!} \left(\frac{k_n}{1-k_n}\right)^{m_n^L - m_n^R} \leq \frac{2}{n} \\
&\Leftrightarrow \quad \frac{n-2}{n} \frac{(n-1-m_n^R)!}{m_n^L!} \frac{m_n^R!}{(n-1-m_n^L)!} \frac{1}{n} \leq \frac{2}{n} \\
&\Leftrightarrow \quad \frac{n-2}{n} \frac{(n-1-m_n^R)!}{m_n^L!} \frac{m_n^R!}{(n-1-m_n^L)!} \leq 2. \quad\quad\quad \text{(B.6)}
\end{aligned}
$$

The first equivalence follows from plugging the definition of $\mu_n(k_n)$ in (B.5) and from simplifying; the second equivalence follows from using the definition of k_n and simplifying. Since the left-hand side of (B.6) converges to 1 as $n \to \infty$, we have shown that the limit belief $\mu = 0$ is consistent with the limit probability $k = \frac{1}{2}$. A similar construction can be used to show that the limit belief $\mu = 1$ is consistent with the limit probability $k = \frac{1}{2}$. $\qquad\square$

Proof of Proposition 3

We first formally introduce the continuation values implied by a Markov strategy and the meaning of optimality under pivotality considerations: If all agents vote according to the Markov strategy $(\lambda(0, p_t), \lambda(1, p_t))$, the decision is

$$
d(p_t) = \begin{cases} L & \text{if } l_t \geq \kappa \\ R & \text{if } l_t \in (1-\kappa, \kappa) \quad \text{with } l_t = p_t \lambda(1, p_t) + (1-p_t)\lambda(0, p_t). \\ R\infty & \text{if } l_t \leq 1-\kappa \end{cases} \quad \text{(B.7)}
$$

Consider the system of linear equations

$$\begin{cases} V_L = \mathbb{E}[\pi_t^i + \delta V_L] \\ V_R = \mathbb{E}[\mathbf{1}_{d(p_t)=L}(\pi_t^i + \delta V_L) + \mathbf{1}_{d(p_t)=R}(\frac{1}{2} + \delta V_R) + \mathbf{1}_{d(p_t)=R\infty}(\frac{1}{2} + \delta V_{R\infty})] \\ V_{R\infty} = \frac{1}{2} + \delta V_{R\infty} \end{cases} \tag{B.8}$$

that is implied by policy (B.7). It has a unique solution. δV_d describes the continuation value from decision $d \in \{R\infty, R, L\}$, common to all agents.

$(\lambda(0, p_t), \lambda(1, p_t))$ is optimal under pivotal voting if and only if, for all $\pi_t^i \in \{0, 1\}$ and all $p_t \in [0, 1]$,

$$\begin{cases} \lambda(\pi_t^i, p_t) = 1 & \text{if } \mu(p_t)(\pi_t^i + \delta V_L) + (1 - \mu(p_t))(\frac{1}{2} + \delta V_R) \\ & \qquad > \mu(p_t)(\frac{1}{2} + \delta V_R) + (1 - \mu(p_t))(\frac{1}{2} + \delta V_{R\infty}) \\ \lambda(\pi_t^i, p_t) \in [0, 1] & \text{if } \mu(p_t)(\pi_t^i + \delta V_L) + (1 - \mu(p_t))(\frac{1}{2} + \delta V_R) \\ & \qquad = \mu(p_t)(\frac{1}{2} + \delta V_R) + (1 - \mu(p_t))(\frac{1}{2} + \delta V_{R\infty}) \\ \lambda(\pi_t^i, p_t) = 0 & \text{if } \mu(p_t)(\pi_t^i + \delta V_L) + (1 - \mu(p_t))(\frac{1}{2} + \delta V_R) \\ & \qquad < \mu(p_t)(\frac{1}{2} + \delta V_R) + (1 - \mu(p_t))(\frac{1}{2} + \delta V_{R\infty}) \end{cases} \tag{B.9}$$

Next, we argue that, in any REMAIN-friendly environment, $\delta^{TM} \in (\delta^*, 1)$. $\delta^{TM} < 1$ is obvious. Since

$$\delta^* < \delta^{TM} \Leftrightarrow \frac{1}{2(1 - \bar{p})} < \frac{1}{1 + F(\frac{1}{2})(1 - 2\bar{p})} \Leftrightarrow F(\frac{1}{2})(1 - 2\bar{p}) < 1 - 2\bar{p},$$

$\bar{p} < \frac{1}{2}$ implies $\delta^* < \delta^{TM}$.

Suppose that the environment is REMAIN-friendly and that $\delta > \delta^{TM}$. We argue in two steps.

Step 1: The optimal policy is implementable. Since $\delta > \delta^{TM}$ implies $\delta > \delta^*$, it is optimal to never LEAVE (by Lemma 1). Thus, if the Markov strategy $(\lambda(0, p_t), \lambda(1, p_t)) = (0, 0)$ is part of an equilibrium, the optimal policy is implementable. This Markov strategy implies $V_R = V_{R\infty}$ (by (B.8)) and $\mu(p_t) = 0$ for all p_t (by (8)). It follows that, for all p_t, an agent believes to be pivotal between REMAIN for now and REMAIN FOREVER; that is, he compares $\frac{1}{2} + V_R$ with $\frac{1}{2} + V_{R\infty}$. Because $V_R = V_{R\infty}$, he is indifferent and $\lambda(\pi_t^i, p_t) = 0$ is for all π_t^i and all p_t

optimal under pivotality considerations for the continuation values and the belief system that it induces.

Step 2: No other policy is implementable. Assume to the contrary that there exists an equilibrium that implements a non-optimal policy. Since the social planner strives to maximize V_R, $V_R < V_R^* = \frac{1}{1-\delta}\frac{1}{2}$ must be true in such an equilibrium. Since it is optimal to never LEAVE, any postponement of LEAVE is socially beneficial. This implies that $V_R \geq V_L$. It follows that

$$0 + \delta V_L < \tfrac{1}{2} + \delta V_R \text{ and } \tfrac{1}{2} + \delta V_R < \tfrac{1}{2} + \delta V_{R\infty}.$$

Hence, no matter in which event a LEAVE-loser believes to be pivotal, he has by (B.9) a strict incentive to vote for REMAIN; that is, only $\lambda(0, p_t) = 0$ is optimal under pivotality considerations.

$\lambda(0, p_t) = 0$ implies $l_t = p_t \lambda(1, p_t) \leq p_t$. For given p_t, we distinguish three cases and argue that, in each case, the decision is not LEAVE. This yields a contradiction to a non-optimal policy being implemented:

Case i: $\lambda(1, p_t) < \frac{1}{2p_t}$. Then, $l_t < \frac{1}{2}$ and, by (8), $\mu(p_t) = 0$. As this means that LEAVE-winners compare $\frac{1}{2} + \delta V_R$ with $\frac{1}{2} + \delta V_{R\infty}$, only $\lambda(1, p_t) = 0$ is optimal under pivotal voting by our observation in the first paragraph of Step 2 and by (B.9). Hence, $d(p_t) = R\infty$.

Case ii: $\lambda(1, p_t) > \frac{1}{2p_t}$. Then, $l_t > \frac{1}{2}$ and only $\mu(p_t) = 1$ is by (8) consistent with the Markov strategy. By (B.9), $\lambda(1, p_t) > 0$ requires $1 + \delta V_L \geq \frac{1}{2} + \delta V_R$. Because, for all $p_t < \frac{1}{2}$, Case i applies, we obtain the following lower bound on V_R:

$$V_R \geq F(\tfrac{1}{2})V_{R\infty} + (1 - F(\tfrac{1}{2}))V_L.$$

Necessary for $1 + \delta V_L \geq \frac{1}{2} + \delta V_R$ is thus

$$
\begin{aligned}
1 + \delta V_L \geq \tfrac{1}{2} + \delta(F(\tfrac{1}{2})V_{R\infty} + (1 - F(\tfrac{1}{2}))V_L) \quad &\Leftrightarrow \quad \tfrac{1}{2} \geq \delta F(\tfrac{1}{2})(V_{R\infty} - V_L) \\
&\Leftrightarrow \quad \tfrac{1}{2} \geq \delta F(\tfrac{1}{2})\tfrac{1}{1-\delta}(\tfrac{1}{2} - \bar{p}) \\
&\Leftrightarrow \quad \delta \leq \delta^{TM}.
\end{aligned}
$$

As this violates our assumption that $\delta > \delta^{TM}$, Case ii cannot occur.

Case iii: $\lambda(1, p_t) = \frac{1}{2p_t}$. Then, $l_t = \frac{1}{2}$ such that $d(p_t) = R$.

Proof of Proposition 4

Suppose that $\delta < \delta^*$. By Lemma 1, a cutoff policy $p^* \in (\frac{1}{2}, 1)$ is optimal. Consider two-sided p^*-majority voting. We will argue that the Markov strategy

$$(\lambda(0, p_t), \lambda(1, p_t)) = \begin{cases} (\frac{1-2p_t}{2-2p_t}, 1) & p_t \le \frac{1}{2} \\ (0, 1) & p_t > \frac{1}{2} \end{cases} \tag{B.10}$$

together with some belief system constitutes an equilibrium. If each agent votes according to this strategy, the mass of LEAVE-votes is $l_t = \max\{\frac{1}{2}, p_t\}$. It follows that, on the equilibrium path, the decision is never $R\infty$ and that LEAVE is chosen in the first period such that $p_t \ge p^*$. That is, the optimal policy is implemented.

It remains to argue that there exists a belief system which is consistent with the Markov strategy, and for which the Markov strategy is optimal. Consider a period t such that $d_{t-1} = R$. We distinguish two cases:

Case 1: $p_t > \frac{1}{2}$. Then, $l_t = p_t$ and, by (8), only $\mu(p_t) = 1$ is consistent with the Markov strategy (B.10). As this means than an agent believes to be pivotal for LEAVE reaching the p^*-majority, he faces the same trade-off as under normal majority voting; i.e., he compares $\pi_t^i + \delta V_L$ with $\frac{1}{2} + \delta V_R$. Since the Markov strategy implies that future decisions are taken according to the optimal policy, we obtain that $\delta(V_R - V_L) = \delta(V_R^* - V_L^*) = \Delta^*$. Since $\Delta^* \in (0, \frac{1}{2})$ for $p^* \in (\frac{1}{2}, 1)$, myopic voting is indeed optimal.

Case 2: $p_t \le \frac{1}{2}$. Then, $l_t = \frac{1}{2}$ and, by (8), any $\mu(p_t) \in [0, 1]$ is consistent with the Markov strategy (B.10). By (B.9), it suffices to argue that there exists some $\mu(p_t) \in [0, 1]$ such that a LEAVE-loser is indifferent between voting for LEAVE and for REMAIN for the continuation values the Markov strategy induces:

$$\mu(p_t)(0 + \delta V_L) + (1 - \mu(p_t))(\tfrac{1}{2} + \delta V_R)$$
$$= \mu(p_t)(\tfrac{1}{2} + \delta V_R) + (1 - \mu(p_t))(\tfrac{1}{2} + \delta V_{R\infty}).$$

Since the Markov strategy (B.10) implies that future decisions are taken according to the

optimal policy, $V_R \geq V_L$ and $V_R \geq V_{R\infty}$ must be true because REMAIN has an option value both relative to LEAVE and relative to REMAIN FOREVER. It follows that

$$0 + \delta V_L < \tfrac{1}{2} + \delta V_R \text{ and } \tfrac{1}{2} + \delta V_R \geq \tfrac{1}{2} + \delta V_{R\infty}.$$

Hence, there exists some $\mu(p_t) \in [0, 1]$ such that the equality holds.

Proofs for Extension I

Derivation of the $(1-\kappa)$-quantile Agent's Stage Payoff, $\pi^{(1-\kappa)}(p_t)$

Define

$$H(\pi|p_t) \equiv \mathrm{Prob}\{\pi_t^i \leq \pi|p_t\} = \begin{cases} (1 - \alpha) - p_t + 2\alpha G(\pi) & \text{if } \pi \in [0, 1) \\ \\ 1 & \text{if } \pi = 1 \end{cases}.$$

We have then $\pi^{(1-\kappa)}(p_t) = 1$ if $\lim_{\pi \to 1} H(\pi|p_t) < 1 - \kappa$ and $\pi^{(1-\kappa)}(p_t) = 0$ if $H(0|p_t) > 1 - \kappa$. Otherwise, $\pi^{(1-\kappa)}(p_t)$ is the unique solution π to the equation $H(\pi|p_t) = 1 - \kappa$. We get

$$\pi^{(1-\kappa)}(p_t) = \begin{cases} 1 & \text{if } p_t > \kappa + \alpha \\ \\ G^{-1}\left(\frac{p_t + \alpha - \kappa}{2\alpha}\right) & \text{if } \kappa - \alpha \leq p_t \leq \kappa + \alpha \\ \\ 0 & \text{if } p_t < \kappa - \alpha \end{cases}. \tag{B.11}$$

Auxiliary Result for the Proof of Lemma 3

Lemma B.5. *(a) If g is quasi-convex, then $\pi \leq G^{-1}(\pi)$ for all $\pi \in [\tfrac{1}{2}, 1]$.*

(b) If g is strictly quasi-concave, then $g(\tfrac{1}{2}) > 1$.

Proof. (a) Quasi-convexity of g and symmetry of g around $\tfrac{1}{2}$ imply that g is weakly increasing on $[\tfrac{1}{2}, 1]$ which in turn implies that G is weakly convex on $[\tfrac{1}{2}, 1]$. It follows from this together with $G(\tfrac{1}{2}) = \tfrac{1}{2}$ and $G(1) = 1$ that, for all $\pi \in [\tfrac{1}{2}, 1]$, $G(\pi) \leq \pi$ and thus, $\pi \leq G^{-1}(\pi)$.

(b) Strict quasi-concavity of g and symmetry of g around $\tfrac{1}{2}$ imply that g is non-constant and attains its maximum at $\tfrac{1}{2}$. As the maximum of a non-constant density with support $[0, 1]$ must be larger than 1, we obtain $g(\tfrac{1}{2}) > 1$. $\qquad\square$

Proof of Lemma 3

(a) Suppose that g is quasi-convex. If $p_t \in (\frac{1}{2} + \alpha, 1 - \alpha]$, (A.1) holds because $\pi^{(1/2)}(p_t) = 1$. It remains to consider the case with $p_t \in (\frac{1}{2}, \min\{\frac{1}{2} + \alpha, 1 - \alpha\}]$. We need to show that $G^{-1}(\frac{p_t + \alpha - 1/2}{2\alpha}) > p_t$. By Lemma B.5 (a), $\frac{p_t + \alpha - 1/2}{2\alpha} > p_t$ is sufficient for this. Since this inequality simplifies to $p_t > \frac{1}{2}$, the desired result follows.

(b) Suppose that g is strictly quasi-concave. (B.11) and symmetry of g around $\frac{1}{2}$ together imply $G^{-1}(\frac{1}{2}) = \frac{1}{2}$ such that $\pi^{(1/2)}(\frac{1}{2}) = \frac{1}{2}$. It suffices thus to show that there exists $\hat{\alpha} \in (0, \frac{1}{2})$ such that for all $\alpha \in (\hat{\alpha}, \frac{1}{2})$, $\frac{\mathrm{d}}{\mathrm{d}p_t}\pi^{(1/2)}(\frac{1}{2}) < 1$. We have

$$\frac{\mathrm{d}}{\mathrm{d}p_t}\pi^{(1/2)}(\tfrac{1}{2}) = \frac{1}{2\alpha}\frac{1}{g(G^{-1}(\frac{1/2 + \alpha - 1/2}{2\alpha}))} = \frac{1}{2\alpha}\frac{1}{g(\frac{1}{2})}.$$

By Lemma B.5 (b), $g(\frac{1}{2}) > 1$ and the result follows.

Proofs for Extension II

Proof of Lemma 4

By subtracting the first equation in (A.3) from the second equation, multiplying both sides of the resulting equation by δ, and using the definition of $\Delta(\pi_{t-1}^i, p_{t-1})$, we obtain

$$
\begin{aligned}
\Delta(\pi_{t-1}^i, p_{t-1}) &= \delta\mathbb{E}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - \pi_t^i + \Delta(\pi_t^i, p_t))|\pi_{t-1}^i, p_{t-1}] \\
&= \delta\mathbb{E}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - q_{\pi_{t-1}^i}(p_{t-1}, p_t) + q_{\pi_{t-1}^i}(p_{t-1}, p_t)\Delta(1, p_t) \\
&\quad + (1 - q_{\pi_{t-1}^i}(p_{t-1}, p_t))\Delta(0, p_t))|\pi_{t-1}^i, p_{t-1}].
\end{aligned}
\tag{B.12}
$$

The future advantage of REMAIN induced by the same policy in the base model is determined by the equation

$$\Delta = \delta\mathbb{E}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - p_t + \Delta)]. \tag{B.13}$$

Define

$$
\begin{aligned}
\bar{\Delta}(p_{t-1}) &\equiv p_{t-1}\Delta(1, p_{t-1}) + (1 - p_{t-1})\Delta(0, p_{t-1}) \tag{B.14} \\
&= \delta\mathbb{E}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - p_t + p_t\Delta(1, p_t) + (1 - p_t)\Delta(0, p_t))|p_{t-1}] \\
&= \delta\mathbb{E}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - p_t + p_t\Delta(1, p_t) + (1 - p_t)\Delta(0, p_t))] \\
&= \delta\mathbb{E}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - p_t + \bar{\Delta}(p_t))].
\end{aligned}
$$

The equality in the second line follows from using the definition of $\Delta(\pi_{t-1}^i, p_{t-1})$ in (B.12) and from Assumption 1. Since the right-hand side does not depend on p_{t-1}, $\bar{\Delta}(p_{t-1})$ must be constant, say $\bar{\Delta}$. Since $\bar{\Delta}(p_{t-1}) = \bar{\Delta}$ implies that $\bar{\Delta}$ is characterized through the same equation as Δ, i.e., through (B.13), we have shown that $\bar{\Delta}(p_{t-1}) = \Delta$.

It follows from (B.14) with $\bar{\Delta}(p_{t-1}) = \Delta$ that, for each p_{t-1}, either $\Delta(0, p_{t-1}) \leq \Delta \leq \Delta(1, p_{t-1})$ or $\Delta(0, p_{t-1}) \geq \Delta \geq \Delta(1, p_{t-1})$. We conclude the proof of this Lemma by showing that $\Delta(0, p_{t-1}) \geq \Delta(1, p_{t-1})$ for any p_{t-1}.

We have

$$\Delta(0, p_{t-1}) - \Delta(1, p_{t-1})$$

$$\stackrel{(B.12)}{=} \delta\mathbb{E}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - q_0(p_{t-1}, p_t) + q_0(p_{t-1}, p_t)\Delta(1, p_t) + (1 - q_0(p_{t-1}, p_t))\Delta(0, p_t))|p_{t-1}]$$

$$-\delta\mathbb{E}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - q_1(p_{t-1}, p_t) + q_1(p_{t-1}, p_t)\Delta(1, p_t) + (1 - q_1(p_{t-1}, p_t))\Delta(0, p_t))|p_{t-1}]$$

$$= \delta\mathbb{E}[\mathbf{1}_{d(p_t)=R}(q_1(p_{t-1}, p_t) - q_0(p_{t-1}, p_t))|p_{t-1}]$$

$$+\delta\mathbb{E}[\mathbf{1}_{d(p_t)=R}(q_1(p_{t-1}, p_t) - q_0(p_{t-1}, p_t))(\Delta(0, p_t) - \Delta(1, p_t))|p_{t-1}]. \tag{B.15}$$

Since Assumption 2 implies $\mathbb{E}[\mathbf{1}_{d(p_t)=R}(q_1(p_{t-1}, p_t) - q_0(p_{t-1}, p_t))] \geq 0$, we obtain

$$\Delta(0, p_{t-1}) - \Delta(1, p_{t-1})$$

$$\geq \delta\mathbb{E}[\mathbf{1}_{d(p_t)=R}(q_1(p_{t-1}, p_t) - q_0(p_{t-1}, p_t))(\Delta(0, p_t) - \Delta(1, p_t))|p_{t-1}]. \tag{B.16}$$

Substituting t for $t + 1$ in (B.16), we obtain a lower bound on $\Delta(0, p_t) - \Delta(1, p_t)$ that can be used in (B.16) to obtain an even smaller lower bound on $\Delta(0, p_{t-1}) - \Delta(1, p_{t-1})$. We obtain

$$\Delta(0, p_{t-1}) - \Delta(1, p_{t-1})$$

$$\geq \delta^2\mathbb{E}[\left(\prod_{t'=t}^{t+1} \mathbf{1}_{d(p_{t'})=R}(q_1(p_{t'-1}, p_{t'}) - q_0(p_{t'-1}, p_{t'}))\right)(\Delta(0, p_{t+1}) - \Delta(1, p_{t+1}))|p_{t-1}].$$

By repeatedly applying this logic r times, the lower bound becomes

$$\Delta(0, p_{t-1}) - \Delta(1, p_{t-1})$$

$$\geq \delta^{r+1}\mathbb{E}[\left(\prod_{t'=t}^{t+r} \mathbf{1}_{d(p_{t'})=R}(q_1(p_{t'-1}, p_{t'}) - q_0(p_{t'-1}, p_{t'}))\right)(\Delta(0, p_{t+r}) - \Delta(1, p_{t+r}))|p_{t-1}].$$

Since, in each single period, the stage payoff from LEAVE is by at most $\frac{1}{2}$ different from the stage payoff from REMAIN, $\Delta(0, p_t) - \Delta(1, p_t)$ is bounded for given δ. Specifically, we have $|\Delta(0, p_t) - \Delta(1, p_t)| \leq \frac{1}{1-\delta}$. This implies that the lower bound on $\Delta(0, p_{t-1}) - \Delta(1, p_{t-1})$ converges to 0 as $r \to \infty$. Hence, $\Delta(0, p_{t-1}) - \Delta(1, p_{t-1})$ cannot be negative.

Acknowledgements

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